4-1 CIVIL GT-II 1,2,3 UNITS

- Let us consider a condition in which the water table is at a depth 'h' above the failure plane and seepage takes place.
- Since steady seepage takes place parallel to the slope. Hence failure plane is also being subjected to additional pore water pressure of magnitude $\gamma_w h \cos^2 i$ normal to the plane in upward direction.
- Normal stress on failure plane CD,

$$\sigma_n = \sigma_z \cos i - U$$
$$= \gamma_z \cos^2 i - \gamma_w h \cos^2 i$$
$$= \cos^2 i\,(\gamma z - \gamma_w h)$$

and tangential stress,

$$\tau = \sigma_z \sin i = \gamma z \cos i \sin i$$

$$\therefore \quad \text{FOS} = \frac{\tau_f}{\tau} = \frac{\sigma_n \tan\phi}{\tau} = \frac{(\gamma z - \gamma_w h)\cos^2 i \tan\phi}{\gamma z \cos i \sin i}$$

$$\text{FOS} = \left(1 - \frac{\gamma_w h}{\gamma z}\right)\frac{\tan\phi}{\tan i}$$

where, γ = Average unit weight of soil above failure plane.

If $(z - h)$ depth of soil has total unit weight γ and soil below water table will be in saturated condition then,

$$\gamma = \frac{\gamma \cdot (z-h) + \gamma_{sat} \cdot h}{(z-h)+h} = \frac{\gamma \cdot (z-h) + \gamma_{sat} \cdot h}{z}$$

NOTE

If water table rises to ground level, then entire soil become saturated and h becomes z.

$$\therefore \quad \text{FOS} = \left(1 - \frac{\gamma_w \cdot z}{\gamma \cdot z}\right)\frac{\tan\phi}{\tan i} = \left(1 - \frac{\gamma_w}{\gamma_{sat}}\right)\frac{\tan\phi}{\tan i} = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi}{\tan i}$$

$$\therefore \quad \gamma_{sat} \simeq 2\gamma_w$$

Hence, $\text{FOS} \simeq \frac{1}{2}\frac{\tan\phi}{\tan i}$

Example 12.1 An infinitely long slop in dense sand having an inclination of 28° to the horizontal. Compute the factor of safety against shear failure if the angle of internal friction of the soil is 33°.

Solution:

Vertical pressure on CD,

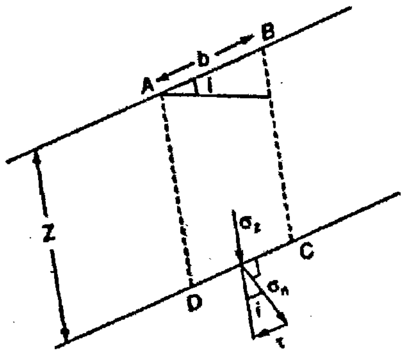
$$\sigma_z = \frac{W}{b \times 1} = \frac{\gamma z b \cos i}{b} = \gamma z \cos i$$

Normal stress on failure plane,

$$\sigma_n = \sigma_z \cos i = \gamma z \cos^2 i$$

and tangential stress,

$$\tau = \sigma_z \sin i$$
$$= \gamma \cdot z \cos i \sin i$$



shear strength of soil on failure plane,

$$\tau_f = C + \sigma_n \tan\phi$$
$$= 0 + \gamma z \cos^2 i \tan\phi$$
$$= \gamma \cdot z \cos^2 i \tan\phi$$

Factor of safety against shear failure,

$$FOS = \frac{\tau_f}{\tau} = \frac{\gamma \cdot z \cos^2 i \cdot \tan\phi}{\gamma z \sin i \cdot \cos i} = \frac{\tan\phi}{\tan i}$$

For given soil, $\phi = 28°$ and $i = 33°$.

$$\therefore \quad FOS = \frac{\tan 33°}{\tan 28°} = 1.22$$

Example 12.2 A granular soil has $G = 2.70$, $e = 0.5$ and $\phi = 35°$. A slope has to be made of this material. If a factor of safety of 1.15 is needed against failure, determine the safe angle of slope when

(a) The slope is dry or submerged without seepage.
(b) If seepage occurs at and parallel to the surface of the slope.
(c) If seepage occurs parallel to the slope with the water table at a depth of 1.5 m, what is the factor of safety available on a slip plane parallel to the ground surface at a depth of 4 m? Assume $i = 30°$.

Solution:

(a) If soil is dry or submerged but no seepage, the factor of safety is given by

$$FOS = \frac{\tan\phi}{\tan i}$$

$$\therefore \quad \tan i = \frac{\tan\phi}{FOS} = \frac{\tan 35°}{1.15} = 0.608$$

$$i = \tan^{-1}(0.608) = 31.36°$$

(b) When the flow occurs at and parallel to the surface of the slope.

$$FOS = \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi}{\tan i}$$

$$\therefore \quad \tan i = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi}{FOS}$$

We know,
$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.70+0.5}{1+0.5}\right) \times 9.81 = 20.928 \text{ kN/m}^2$$

and
$$\gamma' = \gamma_{sat} - \gamma_w = 20.928 - 9.810 = 11.118 \text{ kN/m}^2$$

$$\therefore \quad \tan i = \frac{11.118}{20.928} \times \frac{\tan 35°}{1.15} = 0.323$$

$$i = 17.92°$$

(c) When seepage occurs parallel to the slope with the water table at a depth of 1.5 m, then factor of safety is given by

$$FOS = \left(1 - \frac{\gamma_w h}{\gamma z}\right) \frac{\tan\phi}{\tan i} \quad \text{...(i)}$$

Here, $z = 4$ m and $h = z - 1.5 = 4 - 1.5 = 2.5$ m

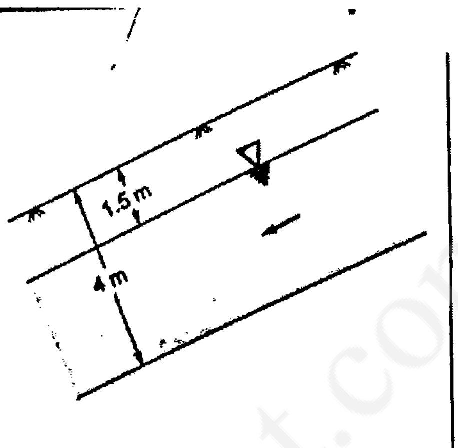
and $\gamma = \dfrac{\gamma_d \times 1.5 + \gamma_{sat} \times 2.5}{1.5 + 2.5}$

where, $\gamma_d = \dfrac{G\gamma_w}{1+e} = \dfrac{2.70 \times 9.81}{1+0.5} = 17.658 \text{ kN/m}^3$

$\therefore \quad \gamma = \dfrac{17.658 \times 1.5 + 20.928 \times 2.5}{1.5 + 2.5} = 19.70 \text{ kN/m}^3$

From equation (i),

$$\text{FOS} = \left(1 - \frac{9.81 \times 2.5}{19.70 \times 4}\right) \cdot \frac{\tan 35^\circ}{\tan 30^\circ} = 0.835$$



2. Cohesive Soils (c–ϕ)

In cohesive soil mass, the shear strength is derived from friction and cohesion both.

$$\tau_f = c + \sigma \tan\phi$$

and factor of safety is given by,

$$\text{FOS} = \frac{\tau_f}{\tau}$$

(a) Dry soil:

$$\therefore \quad \text{FOS} = \frac{\tau_f}{\tau} = \frac{c + \sigma_n \tan\phi}{\tau} = \frac{c + \gamma z \cos^2 i \tan\phi}{\gamma z \cos i \cdot \sin i}$$

- In cohesive soils, if slope angle $i <$ frictional angle ϕ for any given value of normal stress; shear stress will be less than shear strength of soil i.e., $\tau < \tau_f$, which leads to the development of stable slope.
- If slope angle $i >$ frictional angle ϕ, there will be two cases
  - (i) upto the point F, for any value of normal stress, shear stress is still less than the shear strength of soil. Hence slope will be stable.
  - (ii) Beyond point F, if $i > \phi$, but for any given value of normal stress, shear stress exceeds the shear strength of the soil, which leads to the development of unstable slope.

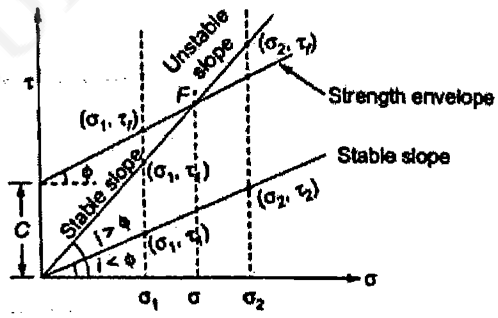


Fig. 12.5

- The height of the slope corresponding to the critical point 'F' is called critical height 'H_c' and factor of safety, FOS = 1

$\therefore$ at $\quad z = H_c$,

$$\text{FOS} = \frac{c + \gamma H_c \cos^2 i \cdot \tan\phi}{\gamma H_c \cos i \cdot \sin i} = 1$$

$$c + \gamma H_c \cos^2 i \gamma \tan\phi = \gamma H_c \cos i \cdot \sin i$$

$$H_c = \frac{c}{\gamma \cos^2 i \cdot (\tan i - \tan\phi)}$$

since, $h = z$ and $\gamma = \gamma_{sat}$

$$\therefore \quad \sigma_n = (\gamma_{sat} - \gamma_w) z \cos^2 i$$

$$FOS = \frac{c + \sigma_n \tan\phi}{\tau} = \frac{c + \gamma' z \cos^2 i \cdot \tan\phi}{\gamma_{sat}\, z \cos i \sin i}$$

12.3 Definitions of Factor of Safety

1. Factor of safety with respect to shear strength is defined as,

$$FOS = \frac{\text{Shear strength of soil } (\tau_f)}{\text{Mobilized shear stress } (\tau)}$$

$$= \frac{C + \bar{\sigma}\tan\phi}{\tau}$$

2. Factor of safety with respect to cohesion,

$$FOS = \frac{c}{c_m}$$

where, c = unit cohesion of soil

c_m = mobilized cohesion of soil

Also, $$FOS = \frac{H_c}{Z}$$

where, H_c = critical depth

$$= \frac{4c}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$$

3. Factor of safety with respect to friction,

$$FOS = \frac{\tan\phi}{\tan\beta}$$

where, ϕ = friction angle of soil

i = slope angle with horizontal

Also, $$FOS = \frac{\tan\phi}{\tan\phi_m}$$

where ϕ_m = mobilized friction angle.

Example 12.3 An infinitely long slope having an inclination of 26° in an area is underlain by firm cohesive soil (G = 2.72 and e = 0.65). There is a thin, weak, layer of soil 6 m below and parallel to the slope surface (c = 25 kN/m², ϕ = 16°). Compute the factor of safety when the slope is dry. If ground water flow could occur parallel to the slope on the ground surface, what factor of safety would result?

Solution:

Given, i = 26°, c = 25 kN/m², ϕ = 16° and z = 6 m

When the slope is dry, the factor of safety is given by the equation

$$FOS = \frac{c + \gamma z \cos^2 i \cdot \tan\phi}{\gamma z \cos i \cdot \sin i}$$

where γ = dry unit weight of soil

$$= \frac{G\gamma_w}{1+e} = \frac{2.72 \times 9.81}{1+0.5} = 17.8 \text{ kN/m}^3$$

$\therefore$

$$FOS = \frac{25 + 17.8 \times 6 \times \cos^2 26° \times \tan 16°}{17.8 \times 6 \times \cos 26° \times \sin 26°}$$

$$= \frac{25 + 24.73}{42.07} = 1.18$$

When there is seepage of water, the factor of safety can be obtained from the following equation,

$$FOS = \frac{c + \gamma' z \cos^2 i \cdot \tan\phi}{\gamma_{sat} \cdot z \cos i \sin i}$$

where

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.72+0.5}{1+0.5}\right)9.81 = 21.05 \text{ kN/m}^3$$

$\therefore$

$$\gamma' = \gamma_{sat} - \gamma_w = 21.05 - 9.81 = 11.25 \text{ kN/m}^3$$

$$FOS = \frac{25 + 11.25 \times 6 \times \cos^2 26° \cdot \tan 16}{21.05 \times 6 \times \cos 26° \times \sin 26°}$$

$$= \frac{40.63}{49.76} = 0.816$$

Example 12.4 A canal having side slopes 1 : 1 is proposed to be constructed in a cohesive soil to a depth of 10 m below the ground surface. The soil properties are $\phi = 15°$, $c = 12$ kPa, $e = 1.0$, $G = 2.65$. Compute the factor of safety with respect to cohesion against failure of the canal bank slope, if Taylor's stability number S_n is 0.08.

Solution:

When the canal flows full, the canal bank slopes will be will submerged under water,

$\therefore$

$$\gamma' = \left(\frac{G_s - 1}{1+e}\right)\gamma_w = \left(\frac{2.65-1}{1+1}\right) \times 9.81 = 8.09 \text{ kN/m}^3$$

We know Taylor's stability no. is given by

$$S_n = \frac{c}{F \cdot \gamma' \cdot H}$$

$$F = \frac{c}{S_n \cdot \gamma' \cdot H} = \frac{12}{0.08 \times 8.09 \times 10} = 1.85$$

12.4 Stability of Finite Slopes

Assumption:

- Failure of the finite slopes takes place due to rotation.
- Soil is homogenous.
- Failure surface is of circular arc shape.

12.4.1 Types of Failure

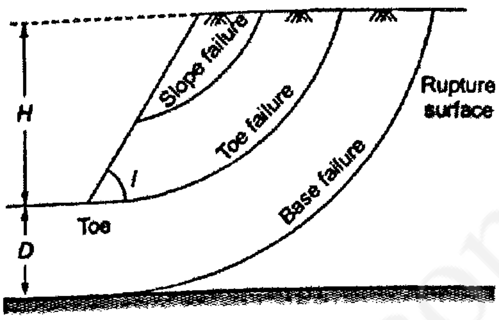


Fig. 12.7

Slope failure:

- If the failure surface intersects with the slope above the toe, it is referred as slope failure.
- This type of failure is observed when soil mass below the toe is strong and soil in upper part is relatively weak.
- ~~Two~~ this type of failure occurs in steep slope.

Toe Failure:

- This type of failure occurs when, failure surface passes through toe.
- This is most common mode of failure which is being observed in finite slope.
- This type of failure occurs when slope is steep ($i > 53°$) and soil above and below the toe is homogenous.

Base Failure:

- This type of failure is observed when the sol mass below the toe is soft and weak in compression and the slope is flat.
- In this case, the failure plane passes below the toe.
- Two type of failure occurs in flat slope.

Method based upon Total Stress analysis:

1. Total stress analysis for purely cohesive soils.
2. Swedish slip circle method.
3. Friction circle method.

12.4.2 Total Stress Analysis for a Purely Cohesive Soil

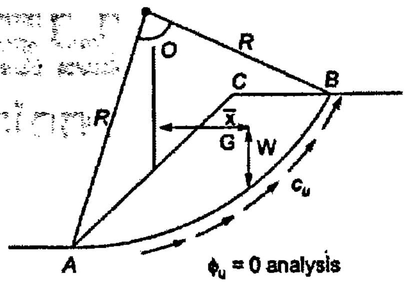


Fig. 12.8

- This is also called '$\phi = 0°$ Analysis'
- This analysis is applied to the case of a newly-built embankment immediately after its construction. (therefore, undrained conditions)
- The slip surface is assumed to be along the arc of a circle, having its centre somewhere above the slope.
- Let W be the weight of the wedge acting vertically downward through G and c_u the unit undrain shear strength of the soil.

Disturbing moment $= W \times \bar{x}$

Shear resistance force along AB = length of Arc AB × unit cohesion

$= (R \cdot \theta)\, c_u$

Resisting moment due to shear $= c_u \cdot R\theta \cdot R = c_u R^2 \cdot \theta$

Thus, factor of safety,
$$F = \frac{\text{Resisting Moment}}{\text{Disturbing Moment}} = \frac{c_u R^2 \theta}{W \bar{x}}$$

Do You Know?

The maximum depth of tension crack is $h_c = \frac{2c}{\gamma}$ and its effect is to shorten the arc along which shearing resistance gets mobilized and thus to reduce central angle θ to θ_D.

The modified $$FOS = \frac{c_u R^2 \theta_D}{W\bar{x} + P_w \cdot l}$$

where, θ_D = changed central angle

Horizontal hydrostatic pressure, $P_W = \frac{1}{2}\gamma_w h_c^2$

12.4.3 Swedish Method of Slices (for c–ϕ soils)

- In this method, the soil mass above the slip plane is divided into a number of slices of equal width.
- The forces acting between these slices is neglected and each slice is assumed to act as an independent column.

  Normal component of weight, $N = W\cos\alpha$ and
  Tangential component of weight, $T = W\sin\alpha$
- Considering the whole slip surface AB of length L.

$$\text{Total driving force} = \Sigma T$$
$$\text{Total resisting forces} = \Sigma cL + \Sigma N\tan\phi = cL + \Sigma N\tan\phi$$

- $$\text{Driving moment} = \Sigma T \cdot R$$
$$\text{Resisting moment} = (cL + \Sigma N\tan\phi)\times R$$

∴ FOS against sliding is given by

$$FOS = \frac{\text{Resisting moment}}{\text{Driving Moment}} = \frac{cL + \Sigma N\tan\phi}{\Sigma T}$$

Since, $N = W\cos\alpha$ and $T = W\sin\alpha$

$$\therefore \quad FOS = \frac{cL + \tan\phi\Sigma W\cos\alpha}{\Sigma W\sin\alpha}$$

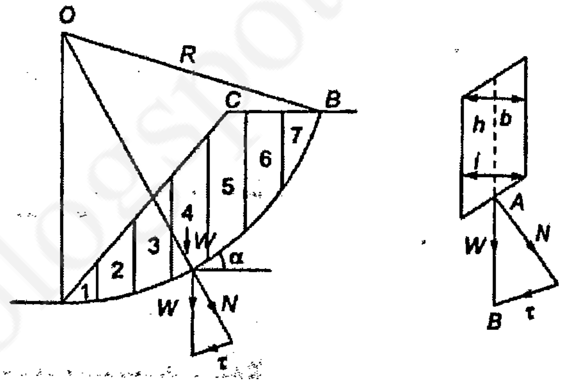


Fig. 12.9

NOTE

- Effect of tension crack is to reduce the arc length, but the depth of crack is given by

$$h_c = \frac{2c}{\gamma}\cdot\tan\left(45° + \frac{\phi}{2}\right)$$

- Partially saturated clays under drain or undrain both gives results similar to that of silt.
- This method is a general approach which is equally applicable to homogenous soils, stratified deposits, partially submerged cases and non uniform slopes.

12.4.4 Friction Circle Method

- In this method, friction angle of the soil is used to analyze the stability of the slope.
- This method is based on the fact that the resultant reaction between the two portions of th soil mass into which trail slip circle divides the slope will be tangential to a concentric smaller circle of radius $r\sin\phi'$.

- Due to tension cracks, the factor of safety against stability reduced.

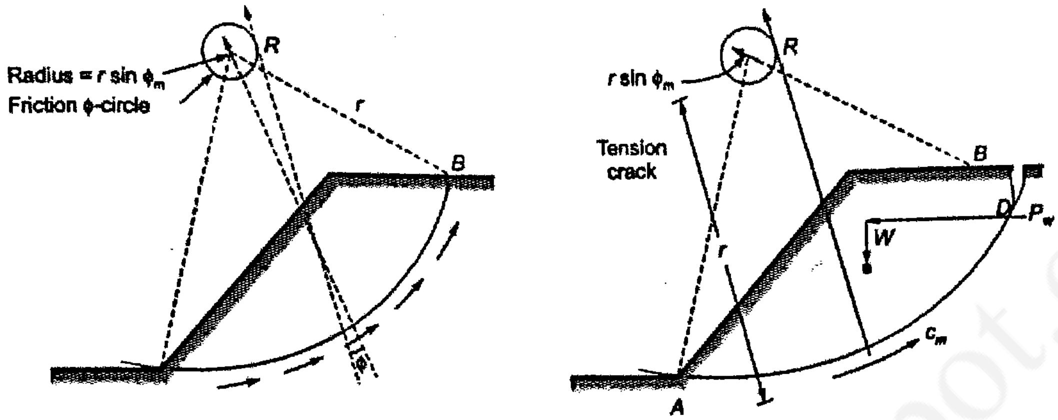


Fig. 12.10 *The friction circle method*

Example 12.4 Stability analysis by the method of slices for 1 : 1 slope on the critical slip gave the following results:

Sum of tangential forces = 150 kN
Sum of normal forces = 320 kN
Sum of neutral forces = 50 kN
Length of failure surface = 18 m
Effective angle of shearing resistance = 15°
Effective cohesion = 20 kN/m²
Calculate the factor of safety with respect to the shear strength.

Solution:

Given, $\Sigma T = 150$ kN
$\Sigma N = 320$ kN
$\Sigma U = 50$ kN

Resultant Normal force, $\Sigma \bar{N} = \Sigma N - \Sigma U = 320 - 50 = 270$ kN/m²

We know, as per Swedish method of slices, factor of safety is given by,

$$FOS = \frac{cL + \Sigma \bar{N} \tan\phi}{\Sigma T}$$

Here, $c = 20$ kN/m², $L = 18$ m
$\phi = 150$

$$\therefore \quad FOS = \frac{20 \times 18 + 270 \times \tan 15^\circ}{150} = 2.88$$

12.5 Effective Stress Analysis

- If pore water pressures exist in an embankment under certain conditions of drainage or seepage, an analysis in terms of effective stress is made.
- This method is applicable at any stage of drainage from no drainage to full drainage.

12.5.1 Steady Seepage

- This condition is encountered in an earth dam or embankment and is considered as critical.
- The effect of pore water pressure is to reduce the effective stress and thereby reduce the stability, because the shear strength mobilized would be decreased.
- The factor of safety is :

$$F = \frac{c'r\theta + \tan\phi'\Sigma(N-u)}{\Sigma T}$$

Where c' and ϕ' are effective stress parameters

ΣU = Total force because of pore pressure

ΣN and ΣT have usual meanings

ΣU can be obtained from flownet. In the absence of flownet following approximate value of F can be used.

$$F = \frac{c'r\theta + \tan\phi'\Sigma N'}{\Sigma T}$$

Here normal components N' of the weights of the slices have to be obtained using submerged unit weight (γ_{sub}) and the Tangential Components T using the saturated unit weight (γ_{sat}).

- Values of F ranging from 1.25 to 1.50 are generally accepted.

12.5.2 Case of Rapid Drawdown

- Sudden or rapid drawdown represents a critical condition since the seepage force in this condition adds to the sliding moment while it reduces the shear resistance mobilized by decreasing the effective stress.
- In this case Factor of safety is calculated by same formula as in case of steady seepage, except, a change in value of u. In this case,

  $u = \gamma_w(h-h')$ instead of $u = \gamma_w \cdot h_w$ (in earlier case)

  where, h_w, h, & h' are as shown in fig.

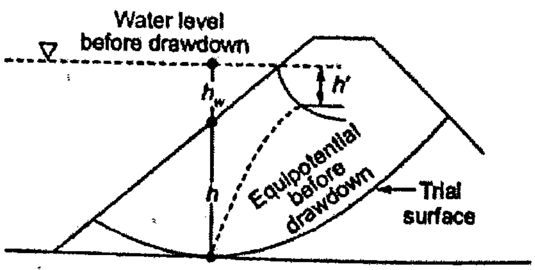


Fig. 12.11

12.5.3 Case of Immediately after Construction

- Rapid construction creates excess pore pressures which affect factor of safety.
- If initial pore pressure is negligible then pore pressure at subsequent stages is given by change in pore pressure which is obtained from Skempton's concept of pore pressure parameters.

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$\frac{\Delta u}{\Delta\sigma_1} = \bar{B} = B\left[\frac{\Delta\sigma_3}{\Delta\sigma_1} + A\left(1 - \frac{\Delta\sigma_3}{\Delta\sigma_1}\right)\right]$$

Now, pore pressure ratio,

$$r_u = \frac{u}{\gamma z} = \frac{\Delta u}{\gamma z} = \frac{\bar{B}.\Delta\sigma_1}{\gamma z}$$

$\Delta\sigma_1$ can be taken nearly equal to the weight of material = γz

$\therefore$ $r_u = \bar{B}$

where, $A, B = ?$

$\bar{B} = ?$

r_u = pore pressure ratio

Now $\bar{B}$ can be obtained from triaxial test. This helps to know pore pressures. Once they are known factor of safety can be obtained in usual manner.

12.6 Effective Stress Analysis by Bishop's Method

- This method takes into account the effect of the forces on the vertical sides of the slices in the Swedish method.
- The procedure is shown below:
- The factor of safety is given by:

$$F = \frac{1}{\Sigma W \sin\alpha} \sum\left[\{c'b + w(1-r_u)\tan\phi'\}\frac{\sec\alpha}{\left(1+\frac{\tan\phi'\tan\alpha}{F}\right)}\right]$$

(Since F is on both sides, solution is by Trial and error.)

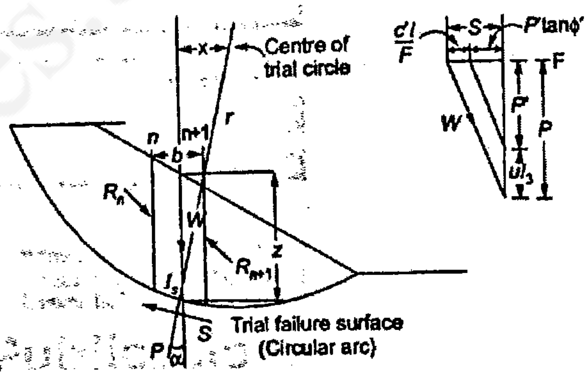


Fig. 12.12

Where, $r_u = \frac{u}{\gamma z}$ = pore pressure ratio

u = pore pressure

W = Weight of slice

α = angle between p and vertical

p = Total Normal force acting on the base of slice.

b = Breadth of slice (= $l_s \cos\alpha$)

c' and ϕ' are effective stress parameters.

Z = height of slice

l_s = length of slice along curved surface.

- Now, if the effect of forces R_n and R_{n+1} is completely ignored and u is expressed in terms of r_u,

$$F = \frac{1}{\Sigma W \sin\alpha} \cdot \Sigma\left[c'l_s + W(\cos\alpha - r_u \sec\alpha)\tan\phi'\right]$$

(which is same as that for steady seepage condition)

12.6.1 Taylor's Stability Number Method

- Taylor introduced a dimensionless parameter, called Taylor's stability Number, which is given by

$$S_n = \frac{c}{\gamma H_c} = \frac{c_m}{\gamma H} = \frac{c}{\gamma F H}$$

Where, H = vertical height of slope

H_c = critical height of slope

c_m = unit cohesion mobilized

F = FOS

- In this method, Taylor's stability number is read from Taylor's stability chart in order to analyze the stability of slope for a given values of strength parameters of the soil.
- Taylor's method is suitable for c-ϕ soils and for $\phi = 0$ (pure clays)

NOTE

- Under dry condition, γ_d and undrain friction angle ϕ_u should be used.
- If soil is fully submerged, the submerged unit weight γ' and effective friction angle (ϕ') should be used.
- If slope is saturated but not submerged, such a case of capillary or sudden drawdown of water table. Then, γ_{sat} should be used along with weighted friction angle of the soil ϕ_w and is given by

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \times \phi$$

where, γ' = submerged unit weight

γ_{sat} = saturated unit weight

ϕ = Friction angle of soil

Example 12.5 A canal having side slopes 1 : 1 is proposed to be constructed in a cohesive soil to a depth of 5 m below ground surface. The soil properties are given below:

$\phi_u = 15°$; $c_u = 12$ kN/m²; $e = 1.0$; and $G = 2.65$

Given, the Taylor's stability No. of soil is 0.08, find the factor of safety with respect to Cohesion against failure of the bank slope.

(i) When canal is full of water

(ii) When there is a sudden drawdown of water in the canal. Use Taylor's stability number (S_n) = 0.126 for $\phi = 5 - 7°$

Solution:

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.65+1.0}{1+1.0}\right) \times 9.81 = 17.9 \text{ kN/m}^3$$

and

$$\gamma' = \gamma_{sat} - \gamma_w = 17.9 - 9.81 = 8.09 \text{ kN/m}^3$$

(a) When canal is running full, the soil will be in submerged condition. Hence γ' will be used.

Here, $S_n = 0.08$, $\gamma' = 8.09$, $c_u = 12$ kN/m²

$$\therefore \quad \text{FOS} = \frac{c_u}{S_n \gamma' H} = \frac{12}{0.08 \times 8.09 \times 5} = 3.7$$

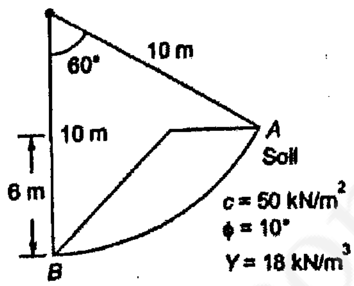
(b) For sudden drawn down, saturated unit weight along with reduced ϕ_w is used.

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \cdot \phi_u = \frac{8.09}{17.9} \times 15 = 6.78°$$

Here, $S_n = 0.126$

$$\therefore \quad \text{FOS} = \frac{c_u}{S_n \gamma_{sat} H} = \frac{12}{0.126 \times 17.9 \times 5} = 1.06$$

Example 12.6 Find the factor of safety (i) with respect to shear strength; and (ii) with respect to height along the indicated sliding surface AB in the figure below.



Normal effective pressure on sliding surface *AB* is 255 kN/m².
Downward tangential disturbing force along *AB* is 840 kN.

Solution:

The total disturbing moment is caused by the tangential disturbing force, given as 840 kN.

The disturbing moment (M_d) = 840 kN × r

$$= 840 \text{ kN} \times 10 \text{ m} = 8400 \text{ kN-m per m run}$$

The resisting moment is provided by the shear strength developed along the failure arc *AB*. Its magnitude

$$= c \cdot \hat{L} + N \tan\phi$$

Normal effective pressure along *AB*

$$= 255 \text{ kN/m}^2 \text{ (given)}$$

Total normal effective force = $N = 255 \times \widehat{AB}$

But $$\widehat{AB} = 2\pi r \cdot \frac{\theta°}{360°} = 2\pi \times 10 \times \frac{60°}{360°} = \frac{20\pi}{6} = 10.47 \text{ m}$$

∴ $$N = 225 \times 10.47 \text{ kN/m run} = 2669.85$$

∴ $$\text{Shearing resistance} = 50\,\frac{\text{kN}}{\text{m}^2} \times 10.47 \text{ m} + 2669.85\,\frac{\text{kN}}{\text{m}} \times \tan 10°$$

$$= 523.5 + 470.8 = 994.3 \text{ kN per m run}$$

Resisting moment

$$= 994.3 \times r = 994.3 \times 10 \text{ m} = 9943 \text{ kN-m/m run} \quad \text{...(i)}$$

(i) FOS w.r.t. shear strength = $\dfrac{M_r}{M_d} = \dfrac{9943 \text{ kN}}{8400 \text{ kN}} = 1.184$

(ii) To calculate FOS with respect to height of the cut, we have to find the max. depth of cut which will be self supporting. This height is given as $2z_0$,

where $$z_0 = \frac{2c}{\gamma \cdot \sqrt{k_a}} \quad \text{...(ii)}$$

∴ Max. depth of unsupported cut $= \dfrac{2 \times 2 \times 50}{18 \times \sqrt{k_a}}$

where $$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 10°}{1+\sin 10°} = \frac{1+0.1736}{1.1736} = 0.704$$

∴ Max depth of cut $= \dfrac{2 \times 2 \times 50}{18 \times \sqrt{0.704}} = 13.24 \text{ m}$

Since the actual cut is of 6 m depth,

Factor of safety available $= \dfrac{13.24 \text{ m}}{6 \text{ m}} = 2.2$

9.4 ILLUSTRATIVE EXAMPLES

Example 9.1: Fig. 9. 27 shows the details of an embankment made of cohesive soil with $\phi = 0$ and $c = 30$ kN/m². The unit weight of the soil is 16.9 kN/m³. Determine the factor of safety against sliding along the trial circle shown. The weight of the sliding mass is 360 kN acting at an eccentricity of 5.0 m from the centre of rotation. Assume that no tension crack develops. The central angle is 70°.

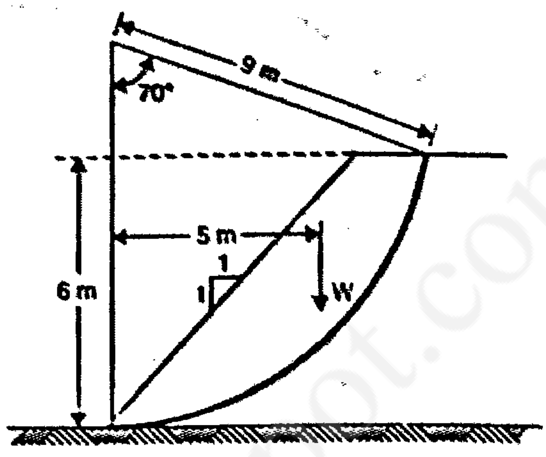


Fig. 9.27 Trial slip circle (Ex. 9.1)

Sliding moment = 360 × 5 = 1800 kNm

Restoring moment

$$= c.\, r^2\theta = 30 \times 9^2 \times \frac{70}{180} \times \pi$$

= 2970 kNm

Factor of safety against sliding,

$F = 2970/1800 = 1.65.$

Example 9.2: A cutting is made 10 m deep with sides sloping at 8 : 5 in a clay soil having a mean undrained strength of 50 kN/m² and a mean bulk density of 19 kN/m³. Determine the factor of safety under immediate (undrained) conditions given the following details of the impending failure circular surface: The centre of rotation lies vertically above the middle of the slope. Radius of failure arc = 16.5 m. The deepest portion of the failure surface is 2.5 m below the bottom surface of the cut (*i.e.*, the centre of rotation is 4 m above the top surface of the cut). Allowance is to be made for tension cracks developing to a depth of 3.5 m from surface. Assume that there is no external pressure on the face of the slope.

(S.V.U.—B.E., (R.R.)—Sep., 1978)

The data are shown in Fig. 9.28.

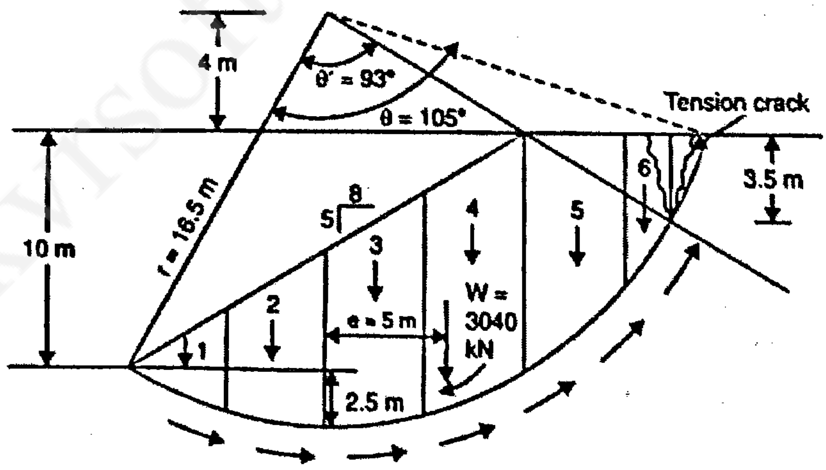


Fig. 9.28 Trial failure surface (Ex. 9.2)

Mean undrained strength = 50 kN/m², $\therefore c = 50$ kN/m²

Factor of safety $F_c = cr^2\theta'/W.\, e$

$= 50 \times 16.5^2 \times (93/180) \times \pi \times 1(3040 \times 5)$

$= 1.45.$

(**Note:** Here, W and e are obtained by dividing the sliding mass into six slices as shown in Fig. 9.28 and by taking moments of the weights of these about the centre of rotation).

Example 9.3: An embankment 10 m high is inclined at an angle of 36° to the horizontal. A stability analysis by the method of slices gives the following forces per running meter:

Σ Shearing forces = 450 kN

Σ Normal forces = 900 kN

Σ Neutral forces = 216 kN

The length of the failure arc is 27 m. Laboratory tests on the soil indicate the effective values c' and ϕ' as 20 kN/m² and 18° respectively.

Determine the factor of safety of the slope with respect to (*a*) shearing strength and (*b*) cohesion.

(*a*) Factor of safety with respect to shearing strength

$$F_s = \frac{c' r\theta + \{\Sigma(N-U)\}\tan\phi'}{\Sigma T}$$

$$= \frac{20 \times 27 + (900 - 216)\tan 18^\circ}{450} = 1.70$$

(*b*) Factor of safety with respect to cohesion

$$F_c = \frac{c' r\theta}{\Sigma T}$$

$$= \frac{20 \times 27}{450} = 1.20.$$

Example 9.4: An embankment is inclined at an angle of 35° and its height is 15 m. The angle of shearing resistance is 15° and the cohesion intercept is 200 kN/m². The unit weight of soil is 18.0 kN/m³. If Taylor's stability number is 0.06, find the factor of safety with respect to cohesion. (S.V.U.—B.Tech. (Part-time)—Apr., 1982)

$\beta = 35°$

$H = 15$ m

$\phi = 15°$

$c = 200$ kN/m²

$\gamma = 18.0$ kN/m³

Taylor's stability Number $N = 0.06$

Since $$N = \frac{c_m}{\gamma H}$$

$\therefore$ $$0.06 = \frac{c_m}{18 \times 15}$$

$\therefore$ Mobilised cohesion,

$$c_m = 0.06 \times 18 \times 15 \text{ kN/m}^2$$

$$= 16.2 \text{ kN/m}^2$$

Cohesive strength $c = 200$ kN/m²

∴ Factor of safety with respect to cohesion:

$$F_c = \frac{c}{c_m} = \frac{200}{16.2} \approx 12.3.$$

Example 9.5: An embankment has a slope of 30° to the horizontal. The properties of the soil are: $c = 30$ kN/m², $\phi = 20°$, $\gamma = 18$ kN/m³. The height of the embankment is 27 m. Using Taylor's charts, determine the factor of safety of the slope.

From Taylor's charts, it will be seen that a slope with $\theta = 20°$ and $\beta = 30°$ has a stability number 0.025. That is to say, if the factor of safety with respect to friction were to be unity (implying full mobilisation of friction), the mobilised cohesion required will be found from

$$N = \frac{c_m}{\gamma H}$$

$$0.025 = \frac{c_m}{18 \times 27}$$

∴ $$c_m = 18 \times 27 \times 0.025 = 12.15 \text{ kN/m}^2$$

∴ Factor of safety with respect to cohesion

$$F_c = c/c_m = 30/12.15 = 2.47$$

But the factor of safety F_s against shearing strength is more appropriate:

$$F_s = \frac{c + \sigma \tan\phi}{\tau}$$

F may be found be successive approximations as follows:

Let us try $F = 1.5$

$$\frac{\tan\phi}{F} = 0.364/1.5 = 0.24267 \approx \text{tangent of angle } 13\frac{2°}{3}$$

With this value of ϕ, the new value N from charts is found to be 0.055.

$$c = 0.055 \times 18 \times 27 = 26.73$$

∴ F with respect to $c = 30/26.73 = 1.12$

Try $F = 1.3$

$$\frac{\tan\phi}{F} = 0.364/1.3 = 0.280 = \text{Tangent of } 15\frac{2°}{3}$$

From the charts, new value of $N = 0.045$

$$c = 0.045 \times 18 \times 27 = 21.87$$

∴ $$F_c = 30/21.87 = 1.37$$

Let us try $F = 1.35$

$$\frac{\tan\phi}{F} = 0.364/1.35 = 0.27 = \text{tangent of angle } 15°.1$$

From the charts, the new value of $N = 0.046$

$$c = 0.046 \times 18 \times 27 = 22.356$$

∴ $$F_c = 30/22.356 = 1.342 \ (= F_\phi)$$

This is not very different from the assumed value.

∴ The factor of safety for the slope = 1.35.

Example 9.6: A cutting is to be made in clay for which the cohesion is 35 kN/m² and $\phi = 0°$. The density of the soil is 20 kN/m³. Find the maximum depth for a cutting of side slope $1\frac{1}{2}$ to 1 if the factor of safety is to be 1.5. Take the stability number for a $1\frac{1}{2}$ to 1 slope and $\phi = 0°$ as 0.17.

(S.V.U.—B.E., (N.R.)—Apr., 1966)

$c = 35$ kN/m² $\phi = 0°$

$\gamma = 20$ kN/m³ $N = 0.17$

$F_c = 1.5$

$$\therefore \quad c_m = c/F_c = 35/1.5 = \frac{70}{3} \text{ kN/m}^2$$

$$\text{But} \quad N = \frac{c_m}{\gamma H}$$

$$\therefore \quad 0.17 = \frac{c_m \times 100}{20 \times H} = \frac{70 \times 100}{3 \times 20 \times H}$$

$$\therefore \quad H = \frac{70 \times 100}{60 \times 017} \text{cm}$$

$$= \mathbf{6.86 \ m.}$$

Example 9.7: At cut 9 m deep is to be made in a clay with a unit weight of 18 kN/m³ and a cohesion of 27 kN/m². A hard stratum exists at a depth of 18 m below the ground surface. Determine from Taylor's charts if a 30° slope is safe. If a factor of safety of 1.50 is desired, what is a safe angle of slope ?

Depth factor $D = 18/9 = 2$

From Taylor's charts,

for $D = 2; \beta = 30°$

$N = 0.172$

$$0.172 = \frac{c_m}{18 \times 9}$$

$c_m = 0.172 \times 18 \times 9 = 27.86$ kN/m²

$c = 27$ kN/m²

$F_c = c/c_m = 27/27.86 = 0.97$

The proposed slope is therefore not safe.

For $F_c = 1.50$

$c_m = c/F_c = 27/1.5 = 18$ kN/m²

$N = 18/18 \times 9 = 1/9 = 0.11$

For $D = 2.0$, and $N = 0.11$

from Taylor's charts,

we have $\beta = 8°$

∴ Safe angle of slope is 8°.

Example 9.8: A canal is to be excavated through a soil with $c = 15$ kN/m², $\phi = 20°$, $e = 0.9$ and $G = 2.67$. The side slope is 1 in 1. The depth of the canal is 6 m. Determine the factor of safety with respect to cohesion when the canal runs full. What will be the factor of safety if the canal is rapidly emptied ?

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.67 \times 0.90}{1+0.90}\right) \times 9.81 \text{ kN/m}^3$$

$$= \frac{3.57}{1.90} \times 9.81 \text{ kN/m}^3 = 18.43 \text{ kN/m}^3$$

$$\gamma' = \gamma_{sat} - \gamma_w = 8.62 \text{ kN/m}^3$$

$$\beta = 45°, \phi = 20°.$$

(*a*) Submerged condition:

From Taylor's charts, for these values of β and φ, the stability number N is found to be 0.06.

$$\therefore \quad 0.06 = \frac{c_m}{\gamma'.H} = \frac{c_m}{8.62 \times 6}$$

$$c_m = 8.62 \times 6 \times 0.06 \text{ kN/m}^2 = 3.10 \text{ kN/m}^2.$$

Factor of safety with respect to cohesion, $F_c = c/c_m = 15/3.10 = 4.48$.

(*b*) Rapid drawdown condition:

$$\phi_w = (\gamma'/\gamma_{sat}) \times \phi = (8.62/18.43) \times 20° = 9.35°$$

For $\beta = 45°$ and $\phi = 9.35°$, Taylor's stability number from charts is found to be 0.114.

$$\therefore \quad 0.114 = \frac{c_m}{\gamma_{sat}H} = \frac{c_m}{18.43 \times 6}$$

$$c_m = 0.114 \times 18.43 \times 6 \text{ kN/m}^2 = 12.60 \text{ kN/m}^2$$

Factor of safety with respect to cohesion $F_c = c/c_m = \dfrac{15.0}{12.6} \approx 1.2$

(Note: The critical nature of a rapid drawdown should now be apparent).

Example 9.9: The cross-section of an earth dam on an impermeable base is shown in Fig. 9.29. The stability of the downstream slope is to be investigated using the slip circle shown. Given:

$$\gamma_{sat} = 19.5 \text{ kN/m}^3$$

$$c' = 9 \text{ kN/m}^2$$

$$\phi' = 27°$$

$$r = 9 \text{ m.}$$

$$\theta = 88°$$

For this circle determine the factor of safety by the conventional approach, as well as the rigorous one.

The line of action of U will pass through the centre of the circle.

The resultant of W and U is the actuating force B.

The triangle of forces will consist of the forces B, C_m and R.

9.3.4 Taylor's Method

For slopes made from two different soils the ratio $c_m/\gamma H$ has been shown to be the same for each slope provided that the two soils have the same angle of friction. This ratio is known as the '*stability number*' and is designated by the symbol, N.

$$\therefore \quad N = c_m/\gamma H \quad \text{...(Eq. 9.43)}$$

where N = stability number (same as S_n of Eq. 9.15)

c_m = Unit cohesion mobilised (with respect to total stress)

γ = Unit weight of soil

and H = Vertical height of the slope (Similar to z of Eq. 9.15).

Taylor (1948) prepared two charts relating the stability number to the angle of slope, based on the friction circle method and an analytical approach. The first is for the general case of a $c-\phi$ soil with the angle of slope less than 53°, as shown in Fig. 9.25. The second is for a soil with $\phi = 0$ and a layer of rock or stiff material at a depth DH below the top of the embankment, as shown in Fig. 9.26. Here, D is known as the depth factor; depending upon its value, the slip circle will pass through the toe or will emerge at a distance nH in front of the toe (the value of n may be obtained from the curves). Theoretically, the critical arc in such cases extends to an infinite depth (slope angle being less than 53°), however, it is limited to the hard stratum. For $\phi = 0$ and a slope angle greater than 53°, the first chart is to be used.

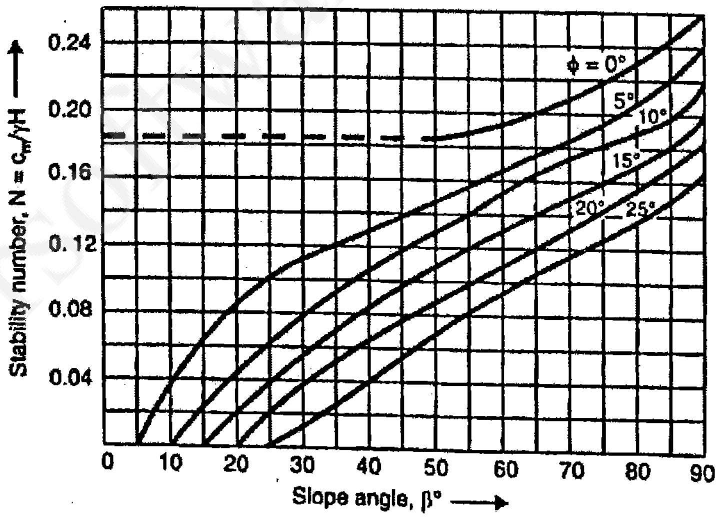


Fig. 9.25 Taylor's charts for slope stability (After Taylor, 1948)
(for $\phi = 0°$ and $\beta < 53°$, use Fig. 9.26)

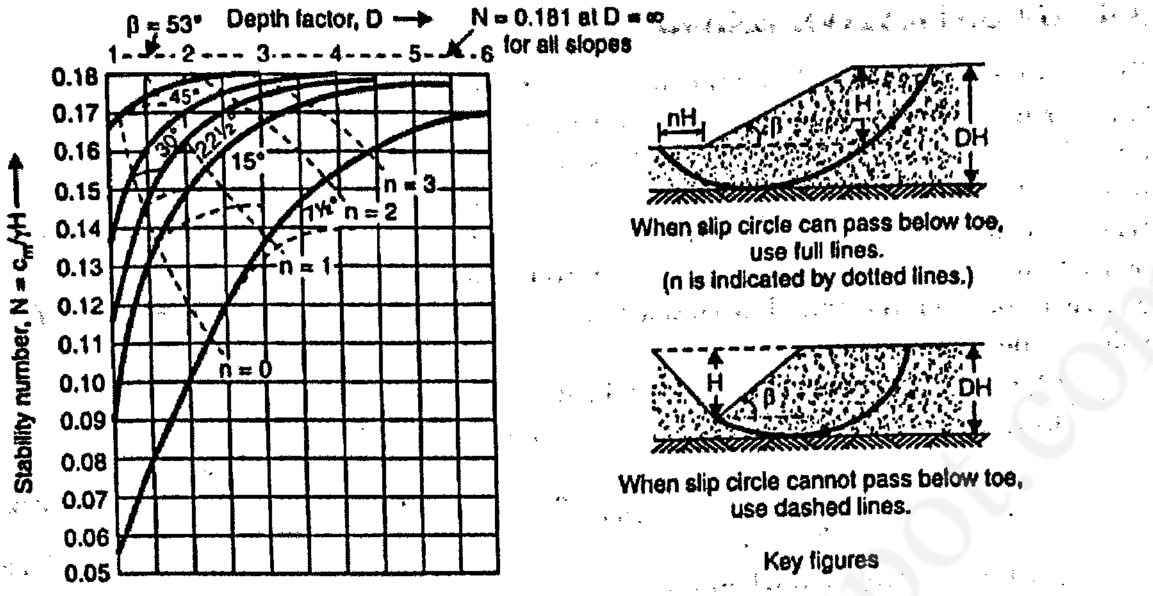


Fig. 9.26 Taylor's chart for slopes with depth limitation (After Taylor, 1948) (for β > 53°, use Fig. 9.25)

(**Note:** For $\phi = 0°$ and $\beta = 90°$, $N = 0.26$. So the maximum unsupported height of a vertical-cut in pure clay is $c/\gamma N$ or $4c/\gamma$ nearly).

The use of the charts is almost self-explanatory. For example, the first chart may be used in one of the two following ways, depending upon the nature of the problem on hand:

1. If the slope angle and mobilised friction angle are known, the stability number can be obtained. Knowing unit weight and vertical height of the slope, the mobilised cohesion can be got.

The factor of safety may be evaluated as the ratio of the effective cohesional strength to the mobilised unit cohesion.

2. Knowing the height of the slope, unit weight of the earth material constituting the slope and the desired factor of safety, the stability number can be evaluated. The slope angle can be found from the chart against the permissible angle of internal friction.

If the slope is submerged, the effective unit weight γ' instead of γ is to be used.

For the case of sudden drawdown, the saturated unit weight γ_{sat} is to be used for γ; in addition, a reduced value of ϕ, ϕ_w, should be used, where:

$$\phi_w = (\gamma'/\gamma_{sat}) \times \phi \qquad \text{...(Eq. 9.44)}$$

Taylor's charts are based on the assumption of full mobilisation of friction, that is, these give the factor of safety with respect to cohesion.

This is all right for a purely cohesive soil; but, in the case of a $c - \phi$ soil, where the factor of safety F_s with respect to shearing strength is desired, ϕ_m should be used for ϕ:

$$\tan \phi_m = \tan \phi / F_s \qquad \text{...(Eq. 9.45)}$$

$$(\text{Also } \phi_m = \phi / F_s)$$

The charts are not applicable for a purely frictional soil ($c = 0$). The stability then depends only upon the slope angle, irrespective of the height of the slope.

$$\therefore \quad r_u = \overline{B} \quad \text{...(Eq. 9.28)}$$

The pore pressure coefficient $\overline{B}$ may be determined from a triaxial test in which the sample is subjected to increases in the principal stresses $\Delta\sigma_1$ and $\Delta\sigma_3$ of magnitudes expected in the field. The resulting pore pressure is measured and $\overline{B}$ is obtained.

Once an idea of pore pressures is got, the factor of safety immediately after construction may be obtained in the usual manner.

Effective Stress Analysis by Bishop's Method

Bishop (1955) gave an effective stress analysis of which he took into account, at least partially, the effect of the forces on the vertical sides of the slices in the Swedish method.

Figure 9.18 illustrates a trial failure surface and all the forces on a vertical slice which tend to keep it in equilibrium.

Let R_n and R_{n+1} be the reactions on the vertical sides of the slice under consideration.

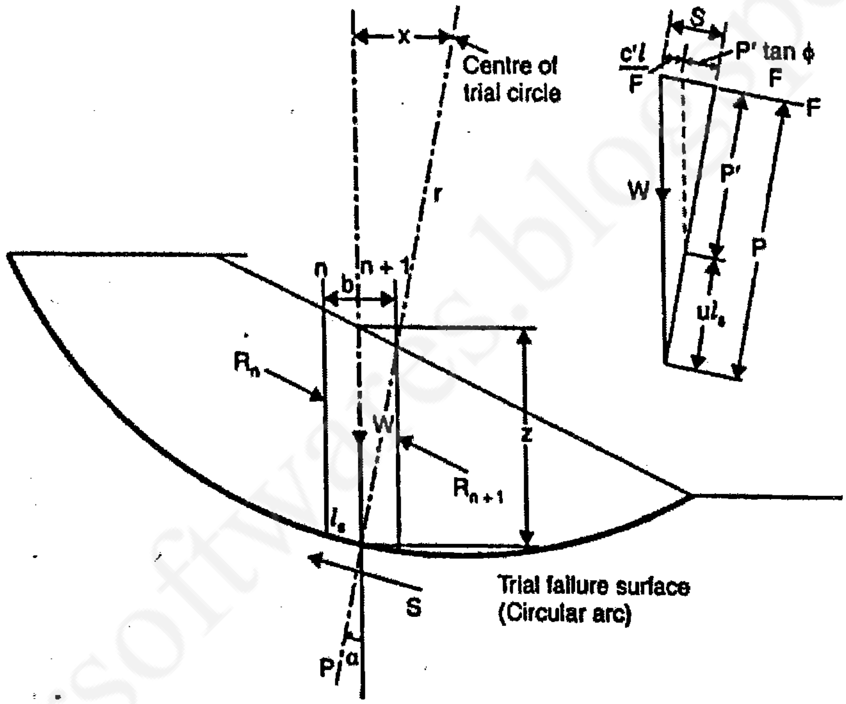


Fig. 9.18 Bishop's procedure for effective stress analysis of slope stability

Let the other forces on the slice be:

W : weight of slice.

P : Total normal force acting on the base of the slice.

S : Shearing resistance acting at the base of slice.

Also, let b : breadth of slice

l_s : length of slice along the curved surface at the base.

z : height of the slice.

x : horizontal distance of the centre of the slice from the centre of the trial slip circle.

α : angle between P and the vertical.

The shear resistance (stress) mobilised is:

$$\tau = \frac{c' + (\sigma_n - u)\tan\phi'}{F}$$

Total normal stress σ_n on the base of the slice = P/l_s

$$\therefore \qquad \tau = \frac{1}{F}[c' + \{(P/l_s) - u\}\tan\phi']$$

Shearing force acting on the base of the slice, $S = \tau l_s$.

For equilibrium,

Sliding moment = Restoring moment

or $$\Sigma W.x = \Sigma S.r = \Sigma\tau.l_s.r$$

$$= \frac{r}{F}\Sigma[c'l_s + (P - ul_s)\tan\phi']$$

$$\therefore \qquad F = \frac{r}{\Sigma Wx}\Sigma[c'l_s + (P - ul_s)\tan\phi'] \qquad \text{...(Eq. 9.29)}$$

Let the normal effective force, $P' = (P - ul_s)$

Resolving the forces vertically,

$$W = P\cos\alpha + S\sin\alpha \qquad \text{...(Eq. 9.30)}$$

(The vertical components of R_n and R_{n+1} are taken to be equal and hence to be nullifying each other, the error from this assumption being considered negligible). Here $P = P' + ul_s$

$$S = 1/F\,(c'l_s + P'\tan\phi')$$

Substituting these values of P and S in Eq. 9.30, we have:

$$W = (P' + ul_s)\cos\alpha + \frac{(c'l_s + P'\tan\phi')\sin\alpha}{F}$$

$$= P'\left(\cos\alpha + \frac{\tan\phi'}{F}\sin\alpha\right) + l_s(u\cos\alpha + (c'/F)\sin\alpha)$$

$$\therefore \qquad P' = \frac{W - l_s(u\cos\alpha + (c'/F)\sin\alpha)}{\cos\alpha + \frac{\tan\phi'\sin\alpha}{F}} \qquad \text{...(Eq. 9.31)}$$

Substituting this value of P' for $(P - ul_s)$ in Eq. 9.29, we get

$$F = \frac{r}{\Sigma Wx}\Sigma\left[\frac{c'l_s + \left(W - ul_s\cos\alpha - \frac{c'l_s}{F}\sin\alpha\right)\tan\phi'}{\left(\cos\alpha + \frac{\tan\phi'\sin\alpha}{F}\right)}\right] \qquad \text{...(Eq. 9.32)}$$

Here $x = r\sin\alpha$

$$b = l_s\cos\alpha$$

$$\frac{ub}{W} = \frac{u}{\gamma z} = r_u$$

Substituting these into Eq. 9.32,

$$F = \frac{1}{\Sigma W\sin\alpha}\Sigma\left[\{c'b + W(1 - r_u)\tan\phi'\}\frac{\sec\alpha}{\left(1 + \frac{\tan\phi'\tan\alpha}{F}\right)}\right] \qquad \text{...(Eq. 9.33)}$$

Since this equation contains F on both sides, the solution should be one by trial and error.

Bishop and Morgenstern (1960) evolved stability coefficients m and n, which depend upon $c'/\gamma H$, ϕ', and cot β. In terms of these coefficients,

$$F = m - nr_u \quad \text{...(Eq. 9.34)}$$

m is the factor of safety with respect to total stresses and n is a coefficient representing the effect of the pore pressures on the factor of safety. Bishop and Morgenstern prepared charts of m and n for sets of $c'/\gamma H$ values and for different slope angles.

If the effect of forces R_n and R_{n+1} is completely ignored, the only vertical force acting on the slice is W.

Hence $P = W \cos \alpha$

$$\therefore \quad F = \frac{r}{\Sigma Wx}\Sigma[c'l_s + (W\cos\alpha - ul_s)\tan\phi']$$

$$= \frac{r}{\Sigma W\sin\alpha}\Sigma[c'l_s + (W\cos\alpha - ul_s)\tan\phi'] \quad \text{...(Eq. 9.35)}$$

since $r \sin \alpha = x$.

If u is expressed in terms of pore pressure ratio r_u,

$$u = r_u\gamma . z = r_u . \frac{W}{b}$$

But $\quad b = l_s \cos \alpha$

$$\therefore \quad u = \frac{r_u W}{l_s \cos\alpha} = \frac{r_u W}{l_s}.\sec\alpha$$

$$\therefore \quad F = \frac{1}{\Sigma W\sin\alpha}\Sigma[c'l_s + W(\cos\alpha - r_u \sec\alpha)\tan\phi'] \quad \text{...(Eq. 9.36)}$$

This is nothing but the Eq. 9.22, obtained by the method of slices and adapted to the case of steady seepage, pore pressure effects being taken into account. This approximate approach is the conventional one while Eq. 9.33 represents the vigorous approach.

9.3.3 Friction Circle Method

The friction circle method is based on the fact that the resultant reaction between the two portions of the soil mass into which the trial slip circle divides the slope will be tangential to a concentric smaller circle of radius $r \sin \phi$, since the obliquity of the resultant at failure is the angle of internal friction, ϕ. (This, of course, implies the assumption that friction is mobilised in full). This can be understood from Fig. 9.19.

This smaller circle is called the 'friction circle' or 'ϕ-circle'.

The forces acting on the sliding wedge are:

(*i*) weight W of the wedge of soil

(*ii*) reaction R due to frictional resistance, and

(*iii*) cohesive force C_m mobilised along the slip surface. These are shown in Fig. 9.20.

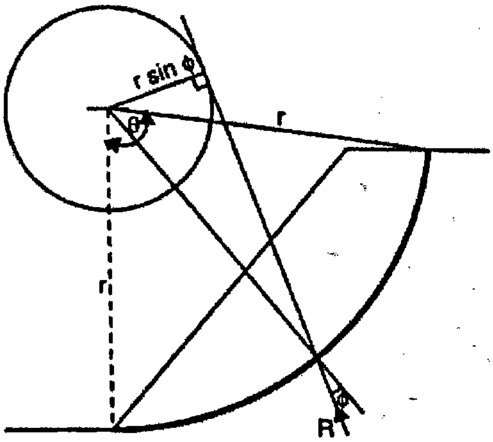


Fig. 9.19 Concept of friction circle

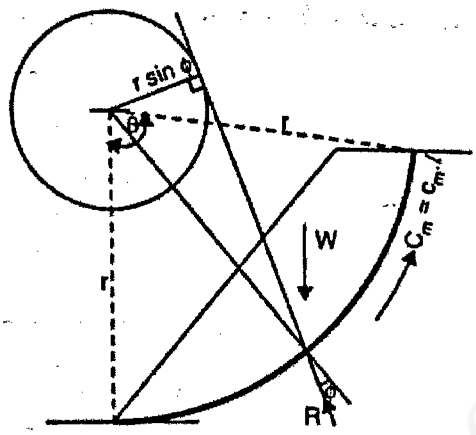


Fig. 9.20 Forces on the sliding wedge

The total reaction *R*, strictly speaking, will not be exactly tangential to the friction circle, but will pass at a slightly greater radial distance than $r \sin \phi$ from the centre of the circular arc. Thus, it can be considered as being tangential to modified friction circle of radius $kr \sin \phi$ where *k* is a constant greater than unity, the value of which is supposed to depend upon the central angle θ and the nature of the distribution of the intergranular pressure along the sliding surface. The concept will be clear if the reactions from small finite lengths of the arc are considered as shown in Fig. 9.21, the total value of *R* being obtained as the vector sum of such small values.

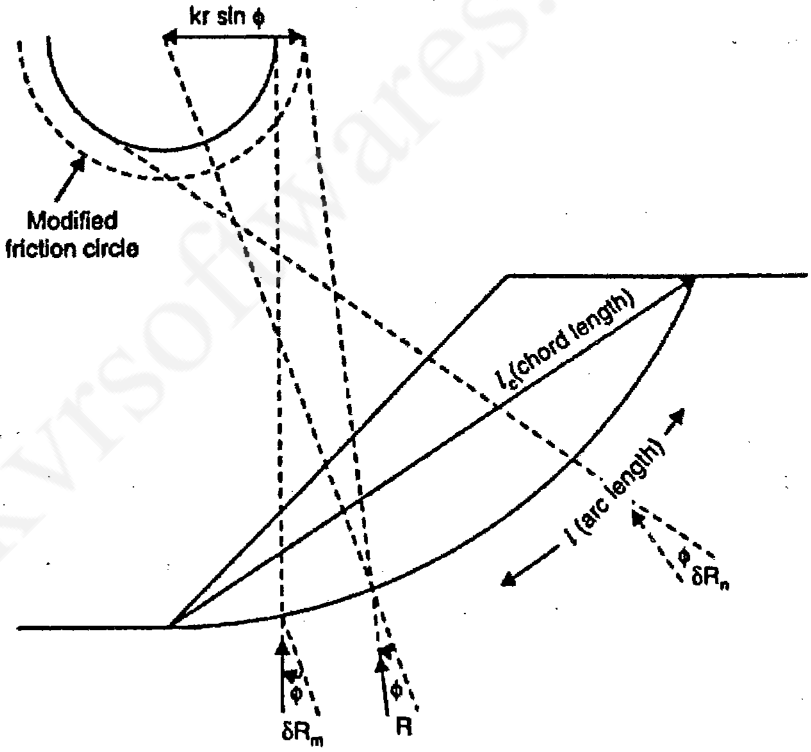
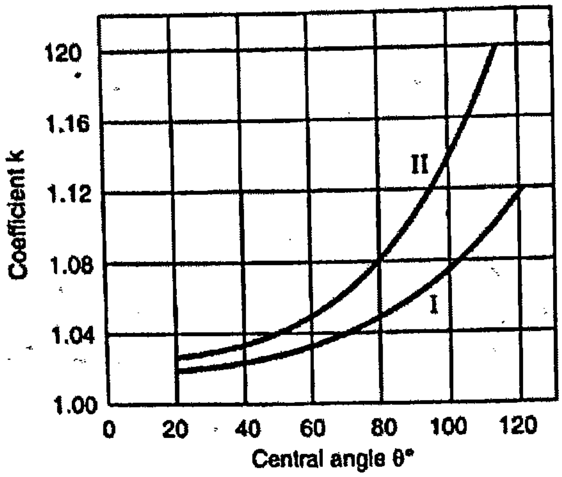


Fig. 9.21 Resultant frictional force-tangential to modified friction circle

The value of k may be obtained from Fig. 9.22.



[*Note:* (1) The relationship I is valid for sinusoidal variation of intergranular pressure with zero values at the ends of the arc, which is considered nearer the actual distribution.
(2) The relationship II is valid for uniform pressure distribution.]

Fig. 9.22 Central angle versus coefficient k for the modified friction circle

Similarly the resultant mobilised cohesive force C_m can be located by equating its moments and the cohesive forces from elementary or finite lengths, into which the whole arc may be divided, about the centre. If c_m is the mobilised unit cohesion, the total mobilised cohesive force all along the arc is $c_m.l$; but the resultant total cohesive force C_m can be shown to be $c_m.l_c$ only where l_c is the chord length since the resultant of an infinite number of small vectors along the arc is the vector along the chord. Putting it in another way, the components parallel to the chord add up to one another while those perpendicular to the chord cancel out on the whole.

Thus, if a is the lever arm of the total cohesive force mobilised, C_m, from the centre of the circle,

$$C_m \cdot a = c_m \cdot l_c . a = c_m . l . r$$

$$a = \frac{l}{l_c} . r \qquad \text{...(Eq. 9.37)}$$

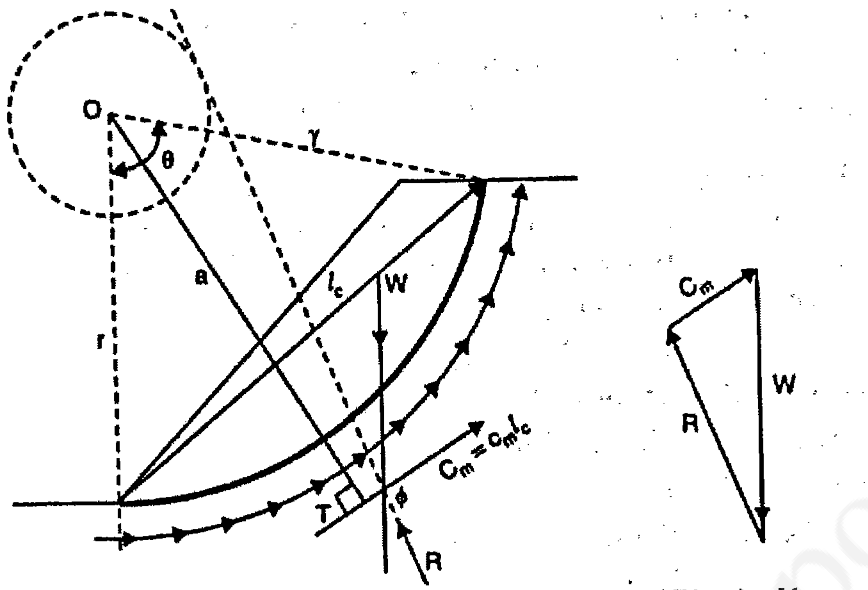
It may be noted that the line of action of C_m does not depend upon the value of C_m.

Fig. 9.23 illustrates these points clearly in addition to showing all the forces and the corresponding triangle of forces.

The lines of action of W and C_m are located first. A tangent is drawn to the modified friction circle from the point of intersection of W and C_m, to give the direction of R. Now the triangle of forces may be completed as shown in Fig. 9.23 (b) drawing W to a suitable scale.

The factor of safety with respect to cohesion, assuming that friction is mobilised in full, is given by:

$$F_c = c/c_m \qquad \text{...(Eq. 9.38)}$$



(a) Resultant cohesive force and other forces (b) Triangle of forces

Fig. 9.23 Location of the resultant cohesive force and triangle of forces

The factor of safety with respect to friction, assuming that cohesion is mobilised in full, is given by:

$$F_\phi = \frac{\tan\phi'}{\tan\phi_m} \quad \text{...(Eq. 9.39)}$$

where ϕ' and ϕ_m are the effective friction angle and mobilised friction angle. If the factor of safety with respect to the total shear strength F_s is required, ϕ_m is to be chosen such that F_c and F_ϕ are equal. This is common-sense and may also be established mathematically:

$$F_s = s/\tau \quad \text{...(Eq. 9.40)}$$

where $s = c' + \bar{\sigma} \tan\phi'$ (shear strength) ...(Eq. 9.41)

and $\tau = c_m + \bar{\sigma} \tan\phi_m$ (shear strength mobilised) ...(Eq. 9.42)

If there were to be neutral pressure due to submergence, it will add to the actuating force as shown in Fig. 9.24.

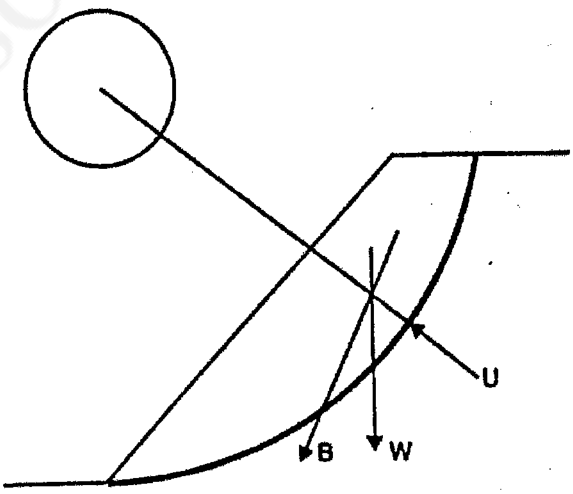


Fig. 9.24 Effect of neutral pressure on the stability of a slope

few earth dams, designed rationally, suffered any damage during earthquakes. However, it is now well understood that ignoring seismic forces is to invite disaster.

Cyclic loads induced by earthquakes decrease the stability of a slope by increasing shear stresses, increasing pore pressures and decreasing the soil strength. Indeed, *liquefaction* is an external manifestation of the decrease in shear strength.

Rigorous approaches are available to study the dynamic response of an earth dam, but they are somewhat complicated and, therefore, are not discussed here. A pseudostatic analysis is one in which the dynamic effect of the earthquake is replaced by a static force. If the maximum acceleration due to earthquake is known, the inertial force on an element of soil is obtained by multiplying the mass of the element with the acceleration. In considering the equilibrium of the sliding mass, this force is included and a factor of safety of more than one is allowed, thus ensuring that no movement of the sliding mass ensues. The inertial force acts only for a brief period in a cycle and lasts for only a few cycles. But the pseudostatic analysis makes no distinction between this transient inertial force and other static forces acting for a much longer duration.

If $k \times g$ is the earthquake acceleration and W is the weight of the slice, the inertial force on it is taken as $k \times W$. The inertial force should be taken at the centroid of the slice, but to simplify the calculations, it is moved down to the base of the slice like the other forces acting on the slice. Fig. 11.18 shown the forces acting on a typical slice. It can be seen that the effect of the inertial force is to add the force $kW \cos\theta$ or $k N$ in the tangential direction while a force $kW \sin\theta$ or $k T$ is decreased from the normal direction. Thus, the expression for the factor of safety now changes to

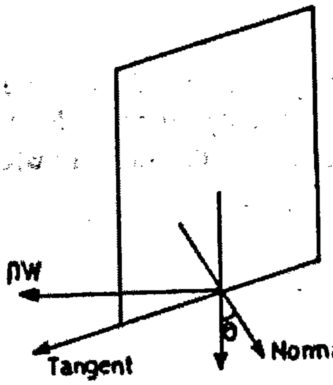


Fig. 11.18 Inertial forces due to earthquake

$$F = \frac{cL + \Sigma(N - U - kT)\tan\phi}{\Sigma(T + kN)} \tag{11.45}$$

A minimum factor of safety of just over one is considered acceptable when considering the effect of earthquake forces. The seismic coefficients commonly used in Japan range from 0.12 to 0.25, depending on the location of the dam, type of foundation and the failure potential downstream of the dam,

EXAMPLES

Example 11.1 An embankment is to be made of a soil which has the following shear strength parameters under the existing conditions:

$$c' = 30 \text{ kN/m}^2 \quad \phi' = 15°$$

If it is assumed that different margins of safety are available for cohesion component and friction component of shearing strength and the mobilised values of cohesion and friction are: $c_m = 22$ kN/m^2, $\phi_m = 12°$, what is the factor of safety with respect to (a) cohesion, and (b) friction?

If the average value of normal effective stress on the failure surface is 120 kN/m^2, what is the value of (a) true factor of safety F_s, (b) F_ϕ when $F_c = 1$ and (c) F_ϕ when $F_\phi = 1$?

Solution:

Given: $c' = 30$ kN/m^2; $\phi' = 15°$

$c_m = 22$ kN/m^2; $\phi_m = 12°$

Factor of safety with respect to cohesion $F_c = \frac{c'}{c_m} = \frac{30}{22} = 1.36$

Factor of safety with respect to friction $F_\phi = \frac{\tan \phi'}{\tan \phi_m} = \frac{\tan 15°}{\tan 12°} = 1.26$

When $\sigma'_f = 120 \text{ kN/m}^2$,

Average shearing strength on the failure surface

$$\tau_f = c' + \sigma'_f \tan \phi' = 30 + 120 \tan 15° = 62.15 \text{ kN/m}^2$$

Average value of shearing stress on the failure surface

$$\tau = c_m + \sigma'_f \tan \phi_m = 22 + 120 \tan 12° = 47.50 \text{ kN/m}^2$$

Thus, factor of safety with respect to shearing strength

$$F_s = \frac{\tau_f}{\tau} = \frac{62.15}{47.50} = 1.31$$

While the factor of safety with respect to shearing strength F_s is 1.31, the value of F_c is 1.36 and $F_\phi = 1.26$. In fact, for the same mobilised shearing resistance τ of 47.50 kN/m^2, there can be so many different combinations of F_c and F_ϕ. The one combination in which the two factors are the same is where $F_c = F_\phi = F_s = 1.31$, which is the true factor of safety.

To find F_ϕ when $F_c = 1$;

$$\tau = 47.5 = 30 + 120 \frac{\tan 15°}{F_\phi}, \text{ giving } F_\phi \text{ a value } 1.84$$

To find F_c when $F_\phi = 1$;

$$\tau = 47.5 = \frac{30}{F_c} + 120 \tan 15° \text{ giving } F_c = 1.95$$

Thus, out of an unlimited combinations of F_c and F_ϕ values that will yield the same mobilised shearing resistance, a few of them are:

F_c	1.00	1.31	1.36	1.95
F_ϕ	1.84	1.31	1.26	1.00

As an alternative to the true factor of safety F_s, the final combination in the above table is often employed in stability analysis. The cohesion required for stability (with full friction mobilised) is directly proportional to the height of the slope—that is, factor of safety with respect to cohesion F_c is equal to the factor of safety with respect to height F_H, and it is the ratio of the critical height to the actual height of the slope, critical height being defined as the maximum height upto which a slope remains stable.

Example 11.2 A granular soil has $\gamma_{sat} = 19 \text{ kN/m}^3$, $\phi' = 35°$. A slope has to be made of this material. If a factor of safety of 1.3 is needed against slope failure, determine the safe angle of the slope (a) when the slope is dry or submerged without seepage, (b) if seepage occurs at and parallel to the surface of the slope. (c) If seepage occurs parallel to the slope with the water table at a depth of 1.5 m, what is the factor of safety available on a slip plane parallel to the ground surface at a depth of 4 m ? Assume $\beta = 28°$.

Solution:

(a) If the soil is submerged but no seepage is occurring, the factor of safety against slippage will be the same as for 'dry' case, as long as there is no change in the value of ϕ'.

From Eq. 11.4,

$$F = \frac{\tan \phi'}{\tan \beta}$$

$$\tan \beta = \frac{\tan 35°}{1.3} = 0.5386$$

$\beta = 28°$ for both dry and submerged soil.

(b) When the flow occurs at and parallel to the ground surface,

$$F = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan \phi'}{\tan \beta}$$

$$\therefore \tan \beta = \left(\frac{19 - 9.8}{19}\right)\frac{\tan 35°}{1.3} = 0.26$$

$$\beta = 14.6°$$

If the slope angle β is kept at 28°, the factor of safety would be reduced to $\left(\frac{9.2}{19}\right)\frac{\tan 35°}{\tan 28°}$, that is, 0.367.

(c) From Eq. 11.5,

$$F = \left(\frac{1 - \gamma_w h}{\gamma_{sat} z}\right)\frac{\tan \phi'}{\tan \beta}$$

$$z = 4 \text{ m};\ h = 4 - 1.5 = 2.5 \text{ m};\ \beta = 28°$$

$$F = \left(1 - \frac{9.8 \times 2.5}{19 \times 4.0}\right)\frac{\tan 35°}{\tan 28°}$$

$$= 0.426$$

Comparison of case (a) with (b) and (c) highlights the influence of pore water pressure on the factor of safety.

Example 11.3 An infinite slope is made of clay with the following properties ; $\gamma_t = 18$ kN/m^3; $\gamma' = 9$ kN/m^3; $c' = 25$ kN/m^2; $\phi' = 28°$. If the slope has an inclination of 35° and height equal to 12m, determine the stability of the slope, when (a) the slope is submerged, and (b) there is seepage parallel to the slope.

Solution:

(a) From Eq. 11.8, using submerged unit weight γ',

$$H_c = \frac{c'}{\gamma'(\tan \beta - \tan \phi')\cos^2 \beta}$$

$$= \frac{25}{9\,(\tan 35^\circ - \tan 28^\circ)\cos^2 35}$$

$$= 24.6 \text{ m}$$

Since the height of the slope H (12 m) is less than the critical height H_c (24.6 m), the slope is stable under submerged condition

(b) From Eq. 11. 8 (a),

$$H_c = \frac{c'}{\gamma_t \left(\tan\beta - \frac{\gamma'}{\gamma_t}\tan\phi'\right)\cos^2\beta}$$

$$= \frac{25}{18\left(\tan 35^\circ - \frac{9}{18}\tan 28^\circ\right)\cos^2 35^\circ}$$

$$= 4.43 \text{ m}$$

Since H (10 m) > H_c (4.43 m), the slope is unstable under condition of seepage parallel to the slope. No slope of height greater than 4. 43 m can stand.

Example 11.4 Fig. 11.19 shows the cross-section of a cutting in a homogeneous, saturated clay soil inclined at a slope of 2 horizontal to 1 vertical, with a height of 8.0 m. Bulk unit weight of the soil is 18 kN/m^3 and undrained unit cohesion is 27 kN/m^2 ($\phi_u = 0$). What is the factor of safety against immediate shear failure along the slip circle shown in Fig. 11.19, (a) ignoring the tension crack, (b) allowing for the tension crack but without water, and (c) allowing for the tension crack filled with water?

Solution:

A 'total stress' analysis is made to determine the factor of safety against immediate shear failure.

(a) Ignoring the tension cracks, let the circular slip surface be AB
Radius of the slip circle, $R = 14.5$ m
Sector angle at centre, $\theta = 121^\circ$
Area of the sliding mass = 110 m^2
Moment arm of weight of sliding wedge = 3.75 m

From Eq. 11.15,
$$F = \frac{c_u R^2 \theta}{Wx} \tag{11.15}$$

$$= \frac{27 \times 14.5^2 \times 121 \times \pi/180}{110 \times 18.0 \times 3.75} = 1.61$$

(b) Development of tension crack reduces the arc length from AB to AD
Depth of tension crack $= \frac{2c_u}{\gamma} = \frac{2 \times 27}{18} = 3$ m
Reduced sector angle of centre, $\theta_D = 109^\circ$
Area of sliding mass = 110 – 1.5 = 108.5 m^2
Centroid distance from O = 3.60 m

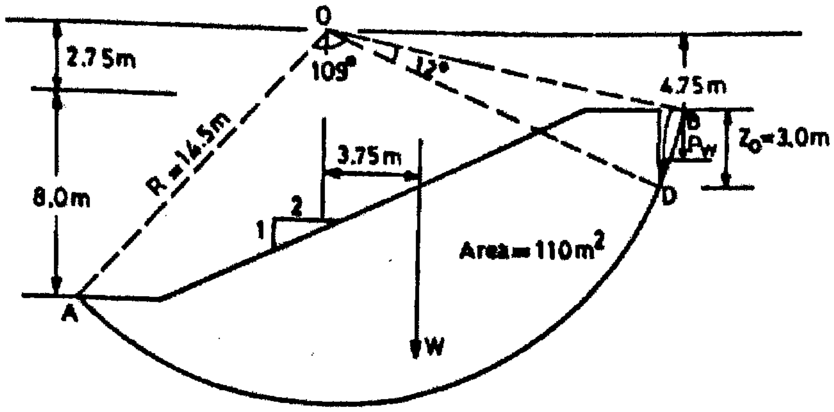


Fig. 11.19 Example 11.4

$$F = \frac{c_u R^2 \theta_D}{Wx} = \frac{27 \times 14.5^2 \times 109 \times \pi/180}{108.5 \times 18.0 \times 3.6} = 1.54$$

(c) When the tension crack is filled with water, the hydrostatic force P_w will cause an additional disturbing moment.

$$P_w = \frac{1}{2}\gamma_w z_o^2 = \frac{1}{2} \times 9.8 \times 3^2 = 44.1 \text{ kN/m}$$

Moment arm of P_w, *i.e.*, $l = 4.75$ m

From Eq. 11. 16,

$$F = \frac{c_u R^2 \theta}{Wx + P_w l} = \frac{27 \times 14.5^2 \times 109 \times \pi/180}{108.5 \times 18 \times 3.6 + 44.1 \times 4.75}$$

$$= 1.49$$

Example 11.5 Determine the factor of safety in terms of effective stress of the slope shown in Fig. 11.20 using the Fellenius method of slices. The relevant soil properties are as follows: $c' = 10$ kN/m^2; $\phi' = 32°$; $\gamma = 20$ kN/m^3

Solution:

The position of the top flow line for the 'steady seepage' condition is shown in Fig. 11.20. This line has been sketched as explained earlier in Chapter 7. The pore pressure at any given point on a final slip surface can be obtained with the help of the equipotential line passing through that point. For example, in Fig. 11.20, the pressure head at point *P* is equal to h_w; that is, the pore pressure *u* is equal to $h_w \gamma_w$. Hence, from the flow net, the pore pressure distribution along the slip surface can be worked out. This will enable the computation of total pore water force acting at the base of each slice as equal to *ul*, where *u* is the pore pressure at the middle of the base and *l*, the length of chord at the base of the slice.

The factor of safety is given by Eq. 11.17 (a), that is,

$$F = \frac{c'L + \tan\phi' \Sigma N'}{\Sigma T}$$

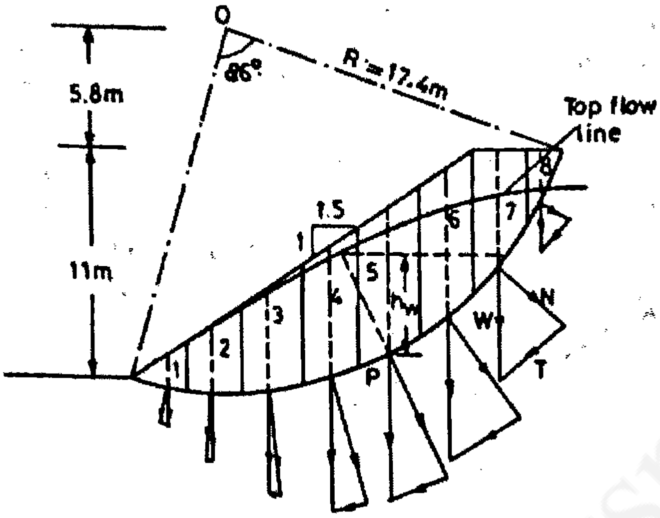


Fig. 11.20 Example 11.5

where $N' = N - ul$.

The soil mass is divided into 8 slices of width equal to 2.06 m. If the average height of each slice is h, the weight of each slice W is given by

$$W = \gamma bh = 20 \times 2.06 \times h = 41.2\, h \text{ kN/m}$$

The weight of each slice (W) is represented by setting off the average height of the slice (h) below the middle of the slice. The normal and tangential components of W (N and T) are determined graphically, as shown in Fig. 11.20. The calculations are tabulated below:

Slice No.	h (m)	W (kN/m)	h_w (m)	L (m)	N (kN/m)	ul (= 9.8 $h_w l$)	N' (kN/m)	T (kN/m)
1.	1.5	61.8	0.9	2.7	60	23.8	36.2	–15
2.	3.3	136	2.1	2.6	136	53.5	82.5	–8
3.	4.8	198	3.9	2.8	196	107	89	31
4.	6.3	260	4.2	3.0	247	123.5	123.5	80.3
5.	6.9	284	4.5	3.3	253	145.5	107.5	129
6.	6.9	284	4.2	3.3	235	135.8	99.2	169
7.	5.4	224.5	2.7	4.2	147	111	36	169
8.	2.4	99	0.6	3.9	49.5	23	26.5	85.7
							$\Sigma N' = 600.4$	$\Sigma T = 631$

Length of the failure surface $L = R \times \dfrac{\theta^\circ}{180} \times \pi$

$$= 17.4 \times \frac{86}{180} \times \pi = 26.12 \text{ m}$$

$$F = \frac{c'L + \tan\phi' \Sigma N'}{\Sigma T}$$

$$= \frac{10 \times 26.12 + \tan 32° \times 600.4}{631} = 1.008$$

Example 11.6 A proposed cutting in a homogeneous cohesive soil will have a slope angle of 25° and will be 8.0 m deep. Using Taylor's stability chart, determine the factor of safety against shear failure in respect of the following soils:

(a) $c_u = 45$ kN/m^2; $\phi_u = 0$; $\gamma = 19$ kN/m^3 ; D is large.

(b) $c_u = 45$ kN/m^2; $\phi_u = 0$; $\gamma = 19$ kN/m^3; The cohesive layer overlies a hard layer of shale, present at a depth of 12 m.

(c) $c_u = 25$ kN/m^2; $\phi_u = 15°$; $\gamma = 18.5$ kN/m^3

Solution:

(a) From Fig. 11.13 (b), for $\phi_u = 0$, $\beta < 53°$ and when D is large, $S_n = 0.181$. When $\phi_u = 0$, factor of safety with respect to shearing strength F_s, is equal to the factor of safety with respect to cohesion F_c.

From Eq. 11.26, $S_n = \dfrac{c_u}{F_c \gamma H}$

Hence, $$F_s = F_c = \frac{c_u}{S_n \gamma H} = \frac{45}{0.181 \times 19 \times 8} = 1.64$$

(b) Depth factor $D = \dfrac{12}{8} = 1.5$.

$$D = 1.5, \beta \text{ (or } i) = 25° \text{ and } \phi_u = 0$$

From Fig. 11.13 (a), $S_n = 0.157$ and $n = 0.3$.

Hence, $$F_s = F_c = \frac{c_u}{S_n \gamma H} = \frac{45}{0.157 \times 19 \times 8} = 1.89$$

(c) $\phi_u = 15°$ and hence, it will be a toe failure.

The true factor of safety will be determined by the method of trials.

Let $F_\phi = 2.0$; $\tan\phi_m = \dfrac{\tan\phi_u}{2} = \dfrac{\tan 15°}{2}$

$$\phi_m = 7.6°$$

For $$\beta = 25°, \phi_m = 7.6°, S_n = 0.068.$$

$$F_c = \frac{c_u}{S_n \gamma H} = \frac{25}{0.068 \times 18.5 \times 8} = 2.48$$

This is not correct, since $F_c > F_\phi$

Let $F_\phi = 2.1$; $$\tan\phi_m = \frac{\tan 15°}{2.1}$$

$$\phi_m = 7.3°.$$

For $\beta = 25°, \phi_m = 7.3, S_n = 0.08$

$$F_c = \frac{25}{0.08 \times 18.5 \times 8} = 2.1 = F_\phi$$

Hence $F_s = 2.1$

Example 11.7 A canal having side slopes 1 to 1 is proposed to be constructed in a cohesive soil to a depth of 5 m below ground surface. The soil properties are given below:

$$\phi_u = 15°; \quad c_u = 12 \text{ kN/m}^2; \quad e = 1.0; \; G_s = 2.65$$

Using Taylor's stability chart, find the factor of safety with respect to cohesion against failure of the bank slopes,

(a) when the canal is full of water and
(b) when there is a sudden drawdown of water in the canal

Solution:

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$= \left(\frac{2.65+1.0}{1+1.0}\right)9.8 = 17.9 \text{ kN/m}^3$$

$$\gamma' = \gamma_{sat} - \gamma_w = 17.9 - 9.8 = 8.1 \text{ kN/m}^3$$

(a) In this case, the submerged unit weight γ' is used.

$$\phi_u = 15°; \beta = 45° \text{ for a } 1:1 \text{ slope}$$

From Fig. 11.13 (b), $S_n = 0.08$

$$F_c = \frac{c_u}{S_n \gamma H} = \frac{12}{0.08 \times 8.1 \times 5} = 3.7$$

(b) For the sudden drawdown case, a reduced value of ϕ is used, according to Eq. 11.27; $\gamma = \gamma_{sat}$.

$$\phi_w = \left(\frac{\gamma'}{\gamma_{sat}}\right)\phi_u = \left(\frac{8.1}{17.9}\right)15 = 6.8°$$

For $\beta = 45°, \phi_w = 6.8°, S_n = 0.126$.

$$F_c = \frac{c_u}{S_n \gamma H} = \frac{12}{0.126 \times 17.9 \times 5} = 1.06$$

Example 11.8 A cutting, 10 m deep, is to be made in a soil having the properties: $\gamma = 19.5$ kN/m^3, $c_u = 20$ kN/m^2, $\phi_u = 15°$. What is the maximum angle of the slope that will have a factor safety against failure of 1.5 ?

Solution:

$F_s = 1.5$ is required, that is, $F_s = F_\phi = F_c = 1.5$.

$\tan\phi_m = \dfrac{\tan 15°}{1.5}$, giving $\phi_m = 10°$

$$F_c = \frac{c_u}{S_n\,\gamma H} = \frac{20}{S_n \times 19.5 \times 10} = \frac{0.1026}{S_n}$$

Trial 1 : $\beta = 15°$.

For $\phi_m = 10°, \beta = 15°, S_n = 0.023$

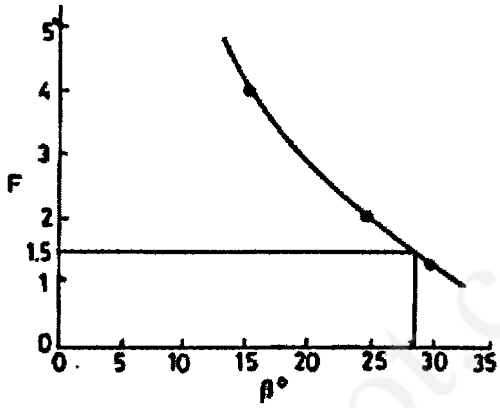


Fig. 11.21 Example 11.8

$$F_c = \frac{0.1026}{0.023} = 4.46$$

Trial 2 : $\beta = 30°$.

$$\text{For } \phi_m = 10°, \beta = 30°, S_n = 0.075$$

$$F_c = \frac{0.1026}{0.075} = 1.37$$

Trial 3 : $\beta = 25°$.

$$\text{For } \phi_m = 10°; \beta = 25°, S_n = 0.049$$

$$F_c = \frac{0.1026}{0.049} = 2.1$$

Plotting β *vs.* F (Fig. 11.21), one can see that for $F = 1.5$, $\beta = 28.5°$

Example 11.9: An excavation is made with a vertical face in a clay soil which has $c_u = 50$ kN/m^2, $\gamma_t = 18$ kN/m^3. Determine the maximum depth of excavation so that the excavation is stable.

Solution:

$\beta = 90°$
For $\phi_u = 0°$ and $\beta = 90°$, $S_n = 0.261$, from Fig. 11.13 (b).
For the maximum depth of stable excavation, $F_c = 1$.

Hence, maximum depth $\quad H = \dfrac{c_u}{S_n F_c \gamma_t}$

$$= \frac{50}{0.261 \times 1 \times 18} = 10.6 \text{ m}$$

A vertical excavation in a clay soil becomes unstable when the stability number becomes less than 0.261.

Example 11.10 A cutting, 20 m deep, is made in a clay soil with its slope at 3 horizontal to 1 vertical. The average value of pore pressure ratio r_u is 0.3. The soil has the following properties:

$$c' = 15 \text{ kN/m}^2;\ \phi' = 20°,\ \gamma = 20 \text{ kN/m}^3$$

Using the Bishop-Morgenstern effective stress stability coefficients, determine the factor of safety of the slope.

Solution:
$$\frac{c'}{\gamma H}=\frac{15}{20\times 20}=0.0375$$

Since there is no chart for this value of $\frac{c'}{\gamma H}$, the interpolation procedure is adopted.

For $c'/\gamma H = 0.025$ and $D = 1.00$ [Fig. 11.16 (d)],

$r_{ue} = 0.55$

Since $r_{ue} > r_u$, use $D = 1.00$ charts.
From the chart [Fig . 11.16 (d)], $m = 1.55$, $n = 1.30$.

$$\therefore\ F_1 = m - nr_u = 1.55 - 1.30\times 0.3 = 1.16$$

For $c'/\gamma H = 0.05$, $D = 1.00$ [Fig. 11.16 (a)],

$r_{ue} = 0$, *i.e.*, $< r_u$

So use the chart with the next higher value of D.

For $\frac{c'}{\gamma H} = 0.05$, $D = 1.25$ [*Fig.*11.16(*b*)],

$r_{ue} > 0.8 > r_u$

Hence, use the $D = 1.25$ charts.
From the chart [Fig. 11.16 (b)],

$$m = 1.80,\ n = 1.40$$

$$\therefore\ F_2 = m - nr_u = 1.80 - 1.40\times 0.3 = 1.38$$

To determine the actual factor of safety value F, interpolate between F_1 and F_2 for $\frac{c'}{\gamma H} = 0.0375$.

$$F = 1.16 + (1.38 - 1.16)\times\frac{0.050 - 0.0375}{0.050 - 0.025}$$

giving $F = 1.27$

PROBLEMS

1. An infinite slope in a sandy soil is inclined at 20° to the horizontal. The properties of the soil are:

$$c' = 0;\ \phi' = 34°,\ \gamma = 17\ \text{kN/m}^3;\ \gamma_{sat} = 21\ \text{kN/m}^3.$$

A hard layer exists 5 m below and parallel to the surface. What is the factor of safety against slip when (a) the slope has negligible water in it, (b) the slope is completely submerged with seepage parallel to the surface, and (c) the water table level is parallel to the ground surface at 2.5 m depth, seepage being parallel. (a) $F = 1.85$; (b) $F = 0.99$ (c) $F = 1.32$

2. An infinite slope is to be constructed of a clay soil at a slope angle of 30°. The ground water level is at the ground surface itself, with seepage parallel to the ground. The soil properties are:

$$c' = 15 \text{ kN/m}^2, \phi' = 22°, \gamma_{sat} = 20 \text{ kN/m}^3$$

What is the factor of safety against movement along a plane parallel to the ground surface at depths of 4 m and 5.5 m? (At $z = 4$ m, $F = 0.79$; At $z = 5.5$ m , $F = 0.67$)

3. Find the critical angle of an infinite slope in a clay soil having $c' = 20$ kN/m^2, $\phi' = 20°$, $G_s = 2.72$ and $e = 0.9$ for the following cases:
   (a) soil is dry,
   (b) the slope is submerged with seepage parallel to the surface.
   A hard stratum exists at a depth of 6 m parallel to the ground surface.
   (a) 40° ; (b) 27.5°

4. A cutting in a clay soil has the section as shown in Fig. 11.22. The shear strength parameters applicable for the soil are:

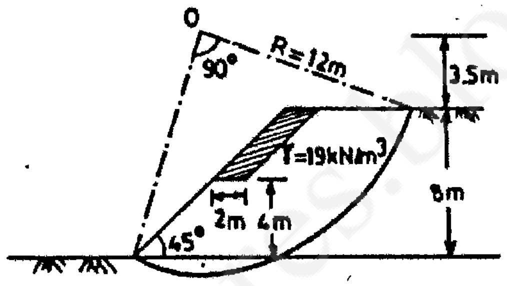


Fig. 11.22 Problem 4

$$c_u = 25 \text{ kN/m}^2 \text{ and } \phi_u = 0$$

The factor of safety of the slope is considered inadequate and hence the section was altered by removing the hatched portion as shown in Fig. 11.22. Determine the factor of safety before and after the proposed alteration.

5. An embankment is made of soil having $c' = 10$ kN/m^2, $\phi' = 23°$ and $\gamma = 19$ kN/m^3. The embankment is of 9 m height and has a slope of 30°. The average pore pressure ratio may be taken as 0.30 for the condition of steady seepage. Using the Fellenius method of slices, determine the factor of safety against shear failure, for a slip circle passing through the toe. Locate the centre of rotation by the Fellenius method. ($F = 1.52$)

6. Using the Bishop's simplified method of slices, determine the factor of safety for problem 5. ($F = 1.83$)

7. If in Problem 5, seepage pressure is not acting, what is the factor of safety associated with the same slip cricle? Assume c', ϕ' and γ to be unchanged. ($F = 2.18$)

8. A canal bank is made of a soil having $c_u = 30$ kN/m,^{2} $\phi_u = 12°$ with $\gamma = 20$ kN/m^3. The canal bank has a height of 10 m, with a slope of 1.5 horizontal to 1 vertical.

To determine the factor of safety of the bank, use any trial slip surface passing through the toe and top of the bank. Using the friction circle method, find the factor of safety of the bank slope with respect to shearing strength by trial and error.

9. A cutting is to be made in a soil deposit for a depth of 15 m in a homogeneous soil with the following properties: $(F = 1.50)$

$$c_u = 25 \text{ kN/m}^2, \phi_u = 18°, \gamma = 18 \text{ kN/m}^3$$

   (a) If a true factor of safety of 1.5 is needed against slope failure, what is the steepest slope angle that can be provided?

   (b) For this slope, what would be the factor of safety with respect to cohesion?

   (a) $\beta = 30°$; (*b*) $F_c = 2.65$

10. An embankment is to be made from a soil with $c_u = 20$ kN/m², $\phi_u = 20°$ and $\gamma = 20$ kN/m³. If a factor of safety of 1.5 with respect to shear strength is required for the embankment slope, determine

    (a) the limiting height of the slope if built at a slope angle of 25°, and

    (b) the seepage angle of the slope if the embankment height is to be kept at 18 m.

    (a) $H = 14.8$ m (b) $\beta = 23°$

11. A cutting 8 m deep is to be made in a saturated clay soil with $\gamma = 20$ kN/m³, $c_u = 28$ kN/m² and $\phi_u = 0$. A hard stratum exists at a depth of 12 m below ground level. Determine the angle of the slope at which failure would occur. If a factor of safety of 1.4 is required, what maximum slope angle can be permitted? (a) $\beta_{max} = 45°$ (b) $\beta = 14°$

12. An embankment of 12 m height with side slopes of 1 : 1 is to be built of a saturated clay, whose properties are as follows:

$$\gamma = 20 \text{ kN/m}^3, c_u = \text{kN/m}^2 \text{ and } \phi_u = 20°$$

    The slope is presenttly completely submerged, but it is anticipated that a 'sudden complete drawdown' may occur in future and the water table may reach below the toe of the slope. Determine

    (a) the factor of safety with respect to height corresponding to the submerged and sudden drawdown cases, and

    (b) the factor of safety with respect to shear strength corresponding to the submerged and sudden drawdown cases, and (a) $F_c = 3.85$; $F_c = 1.136$ (b) $F_s = 2.2$; $F_s = 1.1$

13. A cutting in a clay soil has a slope of 3 horizontal to 1 vertical and has a depth of 16 m. The soil has $c' = 14$ kN/m²; $\phi' = 22°$; $\gamma = 20$kN/m³. The average pore pressure ratio can be taken as 0.2. Using the Bishop-Morgenstern method, obtain the factor of safety for the slope. $(F = 1.56)$

14. A natuaral slope of height 18 m is made at a slope of 2 horizontal to 1 vertical of a uniform saturated clay with $\gamma = 20$ kN/m³, $c' = 9$ kN/m² and $\phi' = 30°$. If the the average pore water pressure ratio r_u on the most critical failure surface is 0.39, determine the factor of safety of the slope by the use of stability coefficients.

    It is desired to increase the factor of safety either by draining the slope or by reducing its inclination.

    If the desired value of factor of safety is 1.40, estimate:

(a) the average value to which r_u must be reduced if the slope profile is left unchanged and

(b) the inclination to which the slope must be trimmed back, assuming r_u to be unchanged.

($F = 1.03$; (a) $r_u = 0.13$ (b) 3 hor. to 1 ver.)

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passive earth resistance plays a major role. Coulomb's theory with curved rupture surfaces, such as the logarithmic spiral, should be used.

For cantilever and counterfort walls, Rankine's theory is used: for gravity and semi-gravity walls, Coulmb's theory is preferred.

13.9 ILLUSTRATIVE EXAMPLES

Example 13.1: A retaining wall, 6 m high, retains dry sand with an angle of friction of 30° and unit weight of 16.2 kN/m³. Determine the earth pressure at rest. If the water table rises to the top of the wall, determine the increase in the thrust on the wall. Assume the submerged unit weight of sand as 10 kN/m³.

(*a*) Dry backfill:

$$\phi = 30° \quad H = 6 \text{ m}$$

$$K_0 = 1 - \sin 30° = 0.5$$

(Also $K_0 = 0.5$ for medium dense sand)

$$\sigma_0 = K_0 \gamma . H$$

$$= \frac{0.5 \times 16.2 \times 600}{1000} \text{N/cm}^2$$

$$= 48.6 \text{ kN/m}^2$$

Thrust per metre length of the wall = $48.6 \times \frac{1}{2} \times 6 = 145.8$ kN

(*b*) Water level at the top of the wall

The total lateral thrust will be the sum of effective and neutral lateral thrusts.

Effective lateral earth thrust, $P_0 = \frac{1}{2} K_0 \gamma . H^2$

$$= \frac{1}{2} \times 0.5 \times 10 \times 6 \times 6 \text{ kN/m.run}$$

$$= 90 \text{ kN/m. run}$$

Neutral lateral pressure $P_w = \frac{1}{2} \gamma_w H^2$

$$= \frac{1}{2} \times 10 \times 6 \times 6 \text{ kN/m.run}$$

$$= 180 \text{ kN/m. run}$$

Total lateral thrust = 270 kN/m. run

Increase in thrust = 124.2 kN/m. run

This represents an increase of about 85.2% over that of dry fill.

Example 13.2: What are the limiting values of the lateral earth pressure at a depth of 3 metres in a uniform sand fill with a unit weight of 20 kN/m³ and a friction angle of 35°? The ground surface is level. (S.V.U.—B.E. (R.R.)—Feb., 1976)

If a retaining wall with a vertical back face is interposed, determine the total active thrust and the total passive resistance which will act on the wall.

Depth, $H = 3$ m

$$\left.\begin{array}{l}\gamma = 20 \text{ kN/m}^3 \\ \phi = 35°\end{array}\right\} \text{for sand fill with level surface.}$$

Limiting values of lateral earth pressure:

Active pressure $= K_a.\gamma H = \dfrac{1-\sin 35°}{1+\sin 35°} \times 20 \times 3$

$= 0.271 \times 60$

$= 16.26$ kN/m^2

Passive pressure $= K_p.\ \gamma H = \dfrac{1+\sin 35°}{1-\sin 35°} \times 20 \times 3$

$= 3.690 \times 60$

$= 221.4$ kN/m^2

Total active thrust per metre run of the wall

$$P_a = \frac{1}{2}\gamma H^2 K_a = 16.26 \times \frac{1}{2} \times 3 = 24.39 \text{ kN}$$

Total passive resistance per metre run of the wall

$$P_p = \frac{1}{2}\gamma H^2.K_p = 221.4 \times \frac{1}{2} \times 3 = 332.1 \text{ kN}$$

Example 13.3: **A gravity retaining wall retains 12 m of a backfill, $\gamma = 17.7$ kN/m^3 $\phi = 25°$ with a uniform horizontal surface. Assume the wall interface to be vertical, determine the magnitude and point of application of the total active pressure. If the water table is a height of 6 m, how far do the magnitude and the point of application of active pressure changed?**

(S.V.U.—Four-year B.Tech—Oct., 1982)

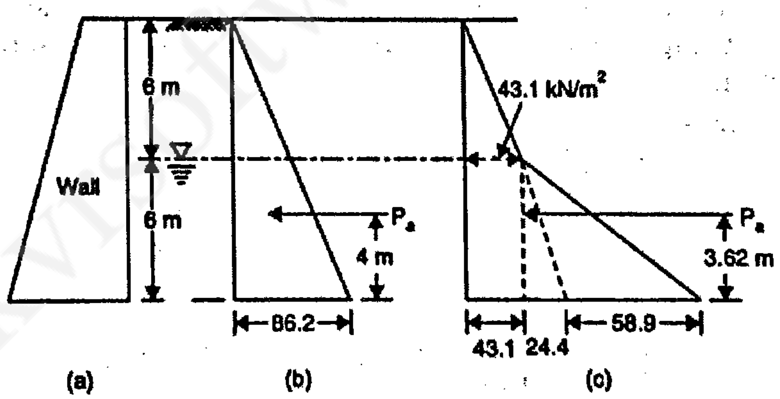


Fig. 13.53 Retaining wall and pressure distribution (Ex. 13.3)

(*a*) Dry cohesionless fill:

$$H = 12 \text{ m} \quad \phi = 25° \quad \gamma = 17.7 \text{ kN/m}^3$$

$$\therefore \quad K_a = \frac{1-\sin 25°}{1+\sin 25°} = 0.406$$

Active pressure at base of wall $= K_a . \gamma H = \frac{1-\sin 25°}{1+\sin 25°} \times 17.7 \times 12$

$= 86.2 \text{ kN/m}^2$

The distribution of pressure is triangular as shown in Fig. 13.53 (*b*).

Total active thrust per metre run of wall $= \frac{1}{2}\gamma H^2 K_a = \frac{1}{2} \times 12 \times 86.2$ = **517.2 kN**

This acts at (1/3)H or 4 m above the base of the wall.

(*b*) Water table at 6 m from surface:

Active pressure at 6 m depth = 0.406 × 17.7 × 6 = 43.1 kN/m²

Active pressure at the base of the wall $= K_a(\gamma . 6 + \gamma' . 6) + \gamma_w .6$

$= 0.406\,(17.7 \times 6 + 10 \times 6) + 9.81 \times 6 = 67.5 + 58.9 = 126.4 \text{ kN/m}^2$

(This is obtained by assuming γ above the water table to be 17.7 kN/m² and the submerged unit weight γ', in the bottom 6 m zone, to be 10 kN/m².

The pressure distribution is shown in Fig. 13.53 (*c*).

Total active thrust per metre run = Area of the pressure distribution diagram

$= \frac{1}{2} \times 6 \times 43.1 + 6 \times 43.1 + \frac{1}{2} \times 6 \times 24.4 + \frac{1}{2} \times 6 \times 58.9$

$= 129.3 + 258.6 + 73.2 + 176.7 = 637.8$ **kN**

The height of its point of application above the base is obtained by taking moments.

$$\bar{z} = \frac{(129.3 \times 8 + 258.6 \times 3 + 73.2 \times 2 + 176.7 \times 2)}{637.8} = \mathbf{3.62 \text{ m}}$$

Total thrust increase by **120.6 kN** and the point of application gets lowered by **0.38 m.**

Example 13.4· A wall, 5.4 m high, retains sand. In the loose state the sand has void ratio of 0.63 and $\phi = 27°$, while in the dense state, the corresponding values of void ratio and ϕ are 0.36 and 45° respectively. Compare the ratio of active and passive earth pressure in the two cases, assuming $G = 2.64$.

(*a*) Loose State:

$G = 2.64 \quad e = 0.63$

$$\gamma_d = \frac{G.\gamma_w}{(1+e)} = \frac{2.64 \times 1}{(1+0.63)} = 16.2 \text{ kN/m}^3$$

$\phi = 27°$

$$K_a = \frac{1-\sin 27°}{1+\sin 27°} 0.376; \; K_p = \frac{1+\sin 27°}{1-\sin 27°} = 2.663$$

Active pressure at depth H m $= K_a . \gamma . H = 0.376 \times 16.2\, H = 6.09.\, H$ kN/m²

Passive pressure at depth H m $= K_p . \gamma H = 2.663 \times 16.2\, H = 43.14\, H$ kN/m²

(*b*) Dense State:

$G = 2.64 \qquad e = 0.36$

$$\gamma_d = \frac{2.64 \times 10}{(1+0.36)} = 19.4 \text{ kN/m}^3$$

$$\phi = 45°$$

$$K_a = \frac{1-\sin 45°}{1+\sin 45°} = 0.172;\ K_p = \frac{1+\sin 45°}{1-\sin 45°} = 5.828$$

Active pressure at depth H m = 0.172 × 19.4H = 3.34 H kN/m^2

Passive pressure at depth H m = 5.828 × 19.4 H = 113.06 H kN/m^2

Ratio of active pressure in the dense state of that in the loose state = $\frac{0.334}{0.609} = 0.55$

Ratio of passive resistance in the dense state to that in the loose state = $\frac{11.306}{4.314} = 2.62$

Example 13.5: A smooth backed vertical wall is 6.3 m high and retains a soil with a bulk unit weight of 18 kN/m^3 and ϕ = 18°. The top of the soil is level with the top of the wall and is horizontal. If the soil surface carries a uniformly distributed load of 4.5 kN/m^2, determine the total active thrust on the wall per lineal metre of the wall and its point of application.

H = 6.3 m γ = 18 kN/m^3 ϕ = 18°

q = 45 kN/m^2

$$K_a = \frac{1-\sin 18°}{1+\sin 18°} = 0.528$$

Active pressure due to weight of soil at the base of wall = $K_a \gamma H$

= 0.528 × 18 × 6.3

= 59.9 kN/m^2

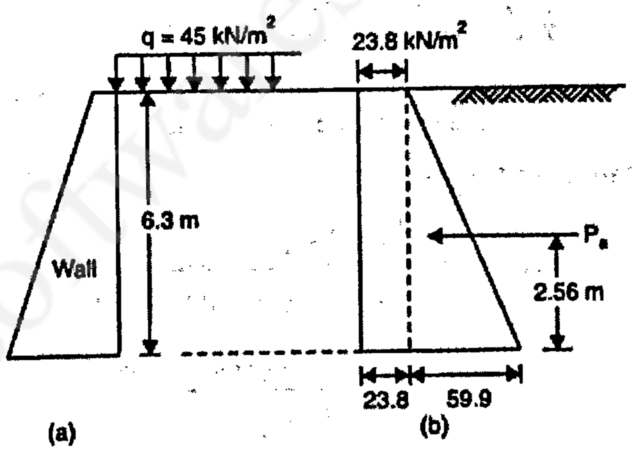


Fig. 13.54 Retaining wall and pressure distribution (Ex. 13.5)

Active pressure due to uniform surcharge = $K_a.q$

= 0.528 × 45

= 23.8 kN/m^2

The former will have triangular distribution while the later will have rectangular distribution with depth. The resultant pressure distribution diagram will be as shown in Fig. 13.54 (*b*).

Total active thrust per lineal metre of wall,

$$P_a = \text{Area of pressure distribution diagram} = \frac{1}{2}K_a\gamma H^2 + K_a qH$$

$$= \frac{1}{2}\times 59.9\times 6.3 + 23.8\times 6.3 = 188.7 + 149.9 = \mathbf{338.6\ kN}$$

The height of its point of application above the base may be obtained by taking moments:

$$\bar{z} = \frac{\left(188.7\times\frac{1}{3}\times 6.3 + 149.9\times\frac{1}{2}\times 6.3\right)}{338.6}\text{m} = \mathbf{2.56\ m}$$

Example 13.6: A vertical wall with a smooth face is 7.2 m high and retains soil with a uniform surcharge angle of 9°. If the angle of internal friction of soil is 27°, compute the active earth pressure and passive earth resistance assuming $\gamma = 20$ kN/m³

$H = 7.2$ m $\quad \beta = 9°$

$\phi = 27° \quad \gamma = 20$ kN/m³

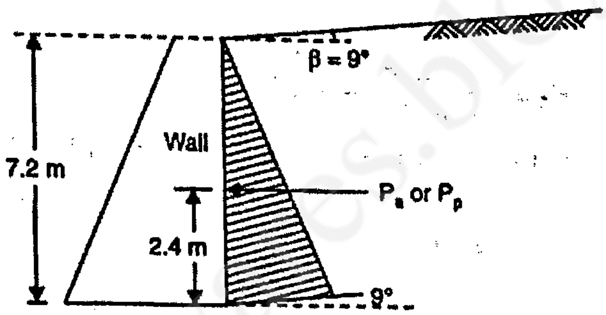


Fig. 13.55 Retaining wall with inclined surcharge and pressure distribution (Ex. 13.6)

According to Rankine's theory,

$$K_a = \cos\beta\left(\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}\right)$$

$$= \cos 9°\left(\frac{\cos 9° - \sqrt{\cos^2 9° - \cos^2 27°}}{\cos 9° + \sqrt{\cos^2 9° - \cos^2 27°}}\right)$$

$$= 0.988 \times 0.397 = 0.392$$

$$K_p = \cos\beta\left(\frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}\right) = 0.988\times\frac{1}{0.397} = 2.488$$

Total active thrust per metre run of the wall

$$P_a = \frac{1}{2}\gamma H^2.K_a = \frac{1}{2}\times 20\times(7.2)^2\times 0.392 = \mathbf{203.2\ kN}$$

Total passive resistance per metre run of the wall

$$P_p = \frac{1}{2}\gamma H^2.K_p = \frac{1}{2} \times 20 \times (7.2)^2 \times 2.488 = \mathbf{1289.8\ kN}$$

The pressure is considered to act parallel to the surface of the backfill soil and the distribution is triangular for both cases. This resultant thrust thus acts at a height of (1/3)*H* or 2.4 m above the base at 9° to horizontal, as shown in Fig. 13.55.

Example **13.7: The Rankine formula of active earth pressure for a vertical wall and a level fill is much better known than the general form and sometimes it is used even when it does not apply. Determine the percentage error introduced by assuming a level fill when the angle of surcharge actually equals 20°. Assume a friction angle of 35° and the wall vertical. Comment of the use of the erroneous result.** (S.V.U.—B.E. (R.R.)—Nov., 1974)

$$\phi = 35°$$

Active pressure coefficient of Rankine for inclined surcharge:

$$K_{ai} = \cos\beta.\left(\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}\right)$$

when $\beta = 0°$ for horizontal surface of the backfill,

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

K_{ai} for $\beta = 20°$ and $\phi = 35°$ is given by

$$K_{ai} = \cos 20°\left(\frac{\cos 20° - \sqrt{\cos^2 20° - \cos^2 35°}}{\cos 20° + \sqrt{\cos^2 20° - \cos^2 35°}}\right) = 0.322$$

K_a for $\beta = 0°$ and $\phi = 35°$ is given by

$$K_a = \frac{1-\sin 35°}{1+\sin 35°} = 0.271$$

Percentage error in the computed active thrust by assuming a level fill when it is actually inclined at 20° to horizontal

$$= \left(\frac{0.322 - 0.271}{0.322}\right) \times 100 = \mathbf{15.84}$$

The thrust is understimated by assuming a level fill, obviously.

Example 13.8: **A retaining wall 9 m high retains a cohesionless soil, with an angle of internal friction 33°. The surface is level with the top of the wall. The unit weight of the top 3 m of the fill is 21 kN/m³ and that of the rest is 27 kN/m³. Find the magnitude and point of application of the resultant active thrust.**

It is assumed that $\phi = 33°$ for both the strata of the backfill.

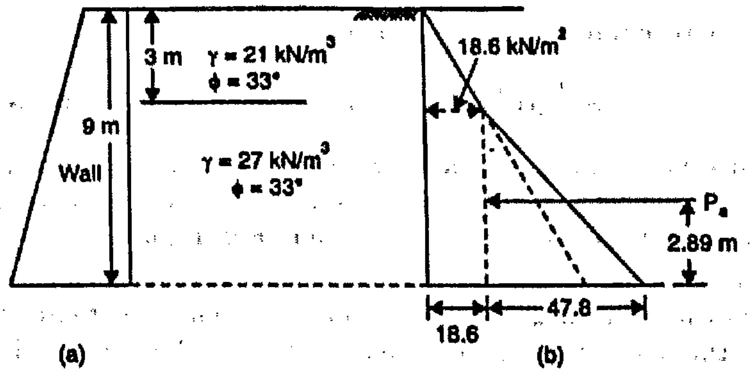


Fig. 13.56 Lateral pressure due to stratified backfill (Ex. 13.8)

$$\therefore \quad K_a = \frac{1 - \sin 33^\circ}{1 + \sin 33^\circ} = 0.295, \text{ for both the strata of the backfill.}$$

Active pressure at 3 m depth

$$K_a \cdot \sigma_v = 0.295\,(21 \times 3) = 18.6 \text{ kN/m}^3$$

Active pressure at the base of the wall

$$K_a \cdot \sigma_v = 0.295\,(21 \times 3 + 27 \times 6) = 66.4 \text{ kN/m}^2$$

The variation of pressure is linear, with a break in the slope at 3 m depth, as shown in Fig. 13.56 (*b*). The total active thrust per metre run, P_a, is given by the area of the pressure distribution diagram.

$$\therefore \quad P_a = \frac{1}{2} \times 3 \times 18.6 + 6 \times 18.6 + \frac{1}{2} \times 6 \times 47.8 = 283 \text{ kN}$$

The height, above the base, of the point of application of this thrust is obtained by taking moments about the base

$$\bar{z} = \frac{(27.9 \times 7 + 111.6 \times 3 + 143.4 \times 2)}{283} \text{ m} = 2.89 \text{ m}$$

Example **13.9: A retaining wall, 7.5 m high, retains a cohsionless backfill. The top 3 m of the fill has a unit weight of 18 kN/m³ and $\phi = 30°$ and the rest has unit weight of 24 kN/m³ and $\phi = 20°$. Determine the pressure distribution on the wall.**

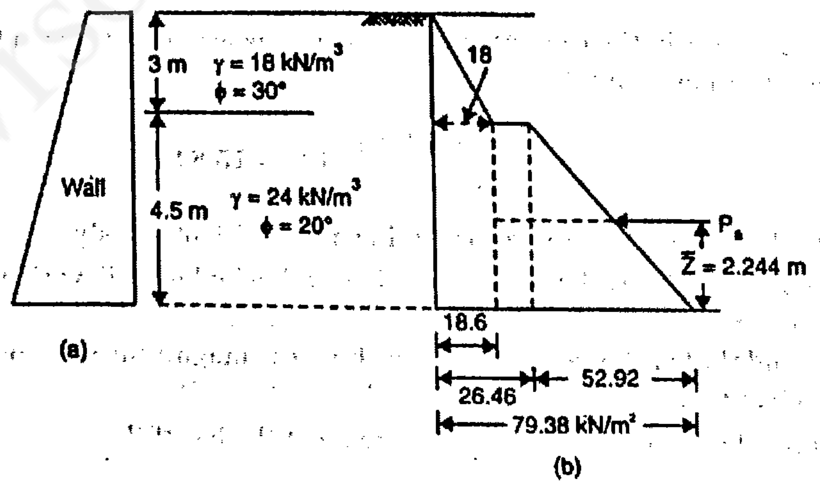


Fig. 13.57 Stratified backfill with different K_a-values for different layers (Ex. 13.9)

$$K_a \text{ for top layer} = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

$$K_a \text{ for bottom layer} = \frac{1-\sin 20^\circ}{1+\sin 20^\circ} = 0.49$$

Active pressure at 3 m depth – considering first layer

$$K_{a_1}.\sigma_v = \frac{1}{3} \times 3 \times 18 = 18 \text{ kN/m}^2$$

Active pressure at 3 m depth – considering second layer

$$K_{a_2}.\sigma_v = 0.49 \times 3 \times 18 = 26.46 \text{ kN/m}^2$$

Active pressure at the base of the wall :

$$K_{a_2} \times 3 \times 18 + K_{a_2} \times 4.5 \times 24 = 26.46 + 0.49 \times 4.5 \times 24 = 79.38 \text{ kN/m}^2$$

The pressure distribution with depth is shown in Fig. 13.57 (*b*).

Total active thrust, P_a, per metre run of the wall

= Area of the pressure distribution diagram

$$= \frac{1}{2} \times 3 \times 18 + 4.5 \times 26.46 + \frac{1}{2} \times 4.5 \times 52.92$$

$$= 27 + 119.07 + 119.07 = 265.14 \text{ kN}$$

The height of the point of application of this thrust above the base of the wall is obtained by taking moments, as usual.

$$\bar{z} = \frac{(27 \times 5.5 + 119.07 \times 2.25 + 11907 \times 1.5)}{265.14} \text{m} = 2.244 \text{ m}$$

Example 13.10: Excavation was being carried out for a foundation is plastic clay with a unit weight of 22.5 kN/m³. Failure occurred when a depth of 8.10 m was reached. What is the value of cohesion if $\phi = 0^\circ$?

$$\phi = 0^\circ \quad \gamma = 22.5 \text{ kN/m}^3$$

Failure occurs when the critical depth, H_c, which is $\frac{4c}{\gamma}.\sqrt{N_\phi}$ is reached.

Since $\phi = 0$, $N_\phi = \tan^2(45^\circ + \phi/2) = 1$

$$\frac{4c}{22.5 \times 1} \times 1 = 8.10$$

$\therefore$ Cohesion, $c = 45.6$ kN/m²

Example 13.11: A sandy loam backfill has a cohesion of 12 kN/m² and $\phi = 20^\circ$. The unit weight is 17.0 kN/m³. What is the depth of the tension cracks ?

Depth of tension cracks, z_c, is given by

$$z_c = \frac{2c}{\gamma}.\sqrt{N_\phi} \quad \phi = 20^\circ$$

$$\sqrt{N_\phi} = \tan(45^\circ + \phi/2) = \tan 55^\circ = 1.428$$

$$c = 12 \text{ kN/m}^2$$
$$\gamma = 17.0 \text{ kN/m}^3$$

$$\therefore \quad z_c = \frac{2 \times 12}{17.0} \times 1.428 \text{ m}$$
$$= 2.00 \text{ m}$$

Example 13.12: **A retaining wall with a smooth vertical back retains a purely cohesive fill. Height of wall is 12 m. Unit weight of fill is 20 kN/m³. Cohesion is 1 N/cm². What is the total active Rankine thrust on the wall? At what depth is the intensity of pressure zero and where does the resultant thrust act?**

$$H = 12 \text{ m} \quad \gamma = 20 \text{ kN/m}^3 \quad \phi = 0°$$
$$N_\phi = \tan^2(45° + \phi/2) = 1$$
$$c = 1 \text{ N/cm}^2 = 10 \text{ kN/m}^2$$
$$z_c = \frac{2c}{\gamma} = \frac{2 \times 10}{20} = 1 \text{ m}$$

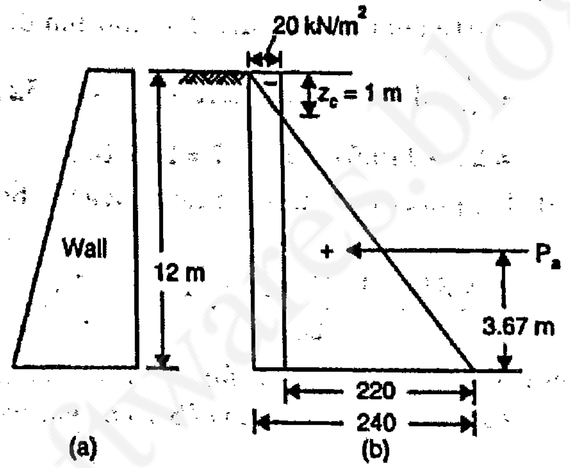


Fig. 13.58 Retaining wall with purely cohesive fill (Ex. 13.12)

∴ The intensity of pressure is zero at a depth of 1 m from the surface.

$$\frac{\gamma H}{N_\phi} = \frac{20 \times 12}{1} = 240 \text{ kN/m}^2$$

$$\frac{2c}{\sqrt{N_\phi}} = \frac{2 \times 10}{\sqrt{1}} = 20 \text{ kN/m}^2$$

The net pressure diagram is shown in Fig. 13.58 (*b*).

The total active thrust may be found by ignoring the tensile stresses, as the area of the positive part of the pressure diagram.

$$P_a = \frac{1}{2} \times 220 \times 11 = 1{,}210 \text{ kN/metre run}$$

This acts at a height of 11/3 m or 3.67 m from the base of the wall.

Example 13.13: **A smooth vertical wall 5 m high retains a soil with $c = 2.5$ N/cm², $\phi = 30°$, and $\gamma = 18$ kN/m³. Show the Rankine passive pressure distribution and determine the magnitude and point of application of the passive resistance.**

$$H = 5 \text{ m} \quad \phi = 30° \quad c = 2.5 \text{ kN/cm}^2 = 25 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

$$N_\phi = \tan^2\left(45° + \frac{30°}{2}\right) = 3$$

Pressure at the base:

$$\gamma H . N_\phi = 18 \times 5 \times 3 = 270 \text{ kN/m}^2$$

$$2c\sqrt{N_\phi} = 2 \times 25 \times \sqrt{3} = 86.6 \text{ kN/m}^2$$

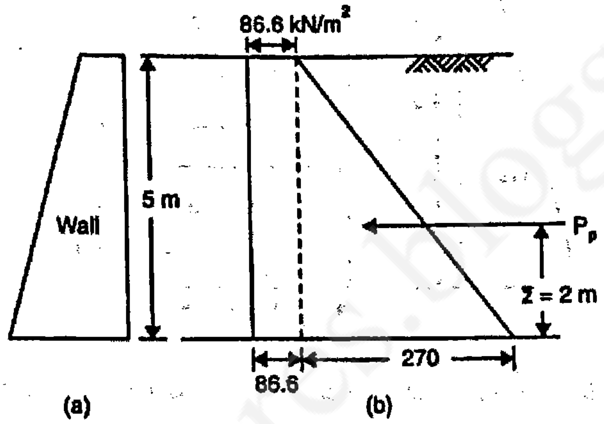


Fig. 13.59 Passive pressure of a $c - \phi$ soil (Ex. 13.13)

The distribution of the first component is triangular and that of the second component is rectangular with depth and the pressure distribution is as shown in Fig. 13.59 (*b*).

The total passive resistance, P_p, on the wall per metre run is obtained as the area of the pressure distribution diagram.

$$\therefore \quad P_p = 5 \times 86.6 + \frac{1}{2} \times 270 \times 5 = 433.0 + 675.0 = \mathbf{1{,}108 \text{ kN}}$$

The height of the point of application above the base is obtained by taking moments as usual.

$$\therefore \quad \bar{z} = \frac{(433 \times 5/2 + 675 \times 5/3)}{1108} \text{ m} = \mathbf{2.00 \text{ m}}$$

Example 13.14: A retaining wall 9 m high retains granular fill weighing 18 kN/m³ with level surface. The active thrust on the wall is 180 kN per metre length of the wall. The height of the wall is to be increased and to keep the force on the wall within allowable limits, the backfill in the top-half of the depth is removed and replaced by cinders. If cinders are used as backfill even in the additional height, what additional height may be allowed if the thrust on the wall is to be limited to its initial value? The unit weight of the cinders is 9 kN/m³. Assume the friction angle for cinders the same as that for the soil.

$H = 9$ m

$\gamma = 18$ kN/m^3

$P_a = 180$ kN/m. run

Initially,

$$P_a = \frac{1}{2}\gamma H^2 . K_a$$

$$\therefore \quad 180 = \frac{1}{2} \times 18 \times 9^2 \times K_a$$

$$K_a = \frac{2 \times 180}{18 \times 9^2} = 0.247$$

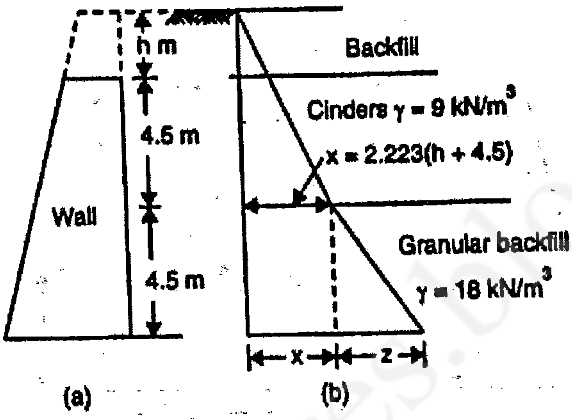


Fig. 13.60 Retaining wall with backfill partly of cinders (Ex. 13.14)

Let the increase in the height of wall be h m.

The depth of cinders backfill will be $(h + 4.5)$ m and bottom 4.5 m is granular backfill with $K_a = 0.247$. Since the friction angles for cinders is taken to be the same as that for the granular soil, K_a for cinders is also 0.247, but γ for cinders is 9 kN/m^3.

The intensity of pressure at $(h + 4.5)$m depth $= 0.247 \times 9\ (h + 4.5)$ kN/m^2

$= 2.223\ (h + 4.5)$ kN/m^2

Intensity of pressure at the base $= 0.247\ [9\ (h + 4.5) + 18 \times 4.5]$ kN/m^2

$= 2.223\ (h + 4.5) + 20$ kN/m^2

Total thrust $P_a' = 1.112\ (h + 4.5)^2 + 2.223 \times 4.5\ (h + 4.5) + \frac{1}{2} \times 4.5 \times 20$

Equating this to the initial value P_a, or 180 kN, the following equation is obtained:

$$1.112\, h^2 + 20h - 67.5 = 0$$

Solving, $h = 2.90$ m

Thus, the height of the wall may be increased by **2.90 m** without increasing the thrust.

Example 13.15: A gravity retaining wall retains 12 m of a backfill. $\gamma = 18$ kN/m^3, $\phi = 30°$ with a uniform horizontal backfill. Assuming the wall interface to be vertical, determine the magnitude of active and passive earth pressure. Assume the angle of wall friction to be 20°. Determine the point of action also. (S.V.U.—Four-year B.Tech.—Dec., 1982)

Since wall friction is to be accounted for, Coulomb's theory is to be applied.

$$\gamma = 18 \text{ kN/m}^3 \text{ and } H = 12 \text{ m}$$

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha.\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

$\alpha = 90°$ and $\beta = 0°$ in this case. $\phi = 30°$ and $\delta = 20°$

$$\therefore \quad K_a = \frac{\cos^2\phi}{\cos\delta\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin\phi}{\cos\delta}}\right]^2}$$

$$= \frac{\cos^2 30°}{\cos 20°\left[1+\sqrt{\frac{\sin 50°.\sin 30°}{\cos 20°}}\right]^2} = 0.132$$

$$K_p = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha.\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\phi+\delta).\sin(\phi+\beta)}{\sin(\alpha+\delta).\sin(\alpha+\beta)}}\right]^2}$$

Putting $\alpha = 90°$ and $\beta = 0°$,

$$K_p = \frac{\cos^2\phi}{\cos\delta\left[1-\sqrt{\frac{\sin(\phi+\delta).\sin\phi}{\cos^2\delta}}\right]^2}$$

$$= \frac{\cos^2 30°}{\cos 20°\left[1-\sqrt{\frac{\sin 50°.\sin 30°}{\cos 20°}}\right]^2} = 2.713$$

$$P_a = \frac{1}{2}\gamma H^2.K_a = \frac{1}{2}\times 18\times 12^2\times 0.132 = 171 \text{ kN/m}$$

$$P_p = \frac{1}{2}\gamma H^2.K_p = \frac{1}{2}\times 18\times 12^2\times 2.713 = 3.516 \text{ kN/m.}$$

Both P_a and P_p act at a height of $(1/3)H$ or 4 m above the base of the wall and are inclined at 20° above and below the horizontal, respectively.

Example 13.16: A retaining wall is battered away from the fill from bottom to top at an angle of 15° with the vertical. Height of the wall is 6 m. The fill slopes upwards at an angle 15° away from the rest of the wall. The friction angle is 30° and wall friction angle is 15°. Using Coulomb's wedge theory, determined the total active and passive thrusts on the wall, per lineal metre assuming $\gamma = 20$ kN/m³.

$$H = 6 \text{ m}$$

$$\beta = 15°$$

$\alpha = 75°$ from Fig. 13.61

$\phi = 30°$
$\delta = 15°$
$\gamma = 20$ kN/m²

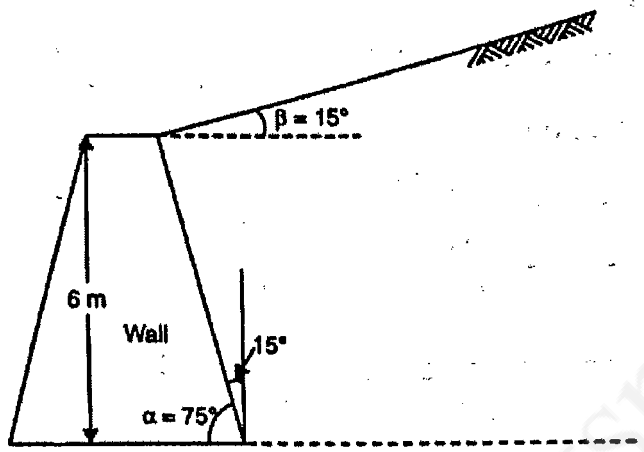


Fig. 13.61 Battered wall with inclined surcharge (Ex. 13.15)

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha.\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

$$= \frac{\sin^2 105°}{\sin^2 75°.\sin 60°\left[1+\sqrt{\dfrac{\sin 45°.\sin 15°}{\sin 60°.\sin 90°}}\right]^2} = 0.542$$

$$K_p = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha.\sin(\alpha+\delta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta).\sin(\phi+\beta)}{\sin(\alpha+\delta).\sin(\alpha+\beta)}}\right]^2}$$

$$= \frac{\sin^2 45°}{\sin^2 75°.\sin 90°\left[1-\sqrt{\dfrac{\sin 45°.\sin 45°}{\sin 90°.\sin 90°}}\right]^2} = 6.247$$

Total active thrust, P_a, per lineal metre of the wall

$$= \frac{1}{2}\gamma H^2.K_a = \frac{1}{2}\times 20\times 6^2\times 0.542 = 195 \text{ kN}$$

Total passive resistance, P_p, per lineal metre of the wall

$$= \frac{1}{2}\gamma H^2.K_p = \frac{1}{2}\times 20\times 6^2\times 6.247 = 2{,}249 \text{ kN}$$

Example 13.17: A vertical retaining wall 10 m high supports a cohesionless fill with $\gamma = 18$ kN/m³. The upper surface of the fill rises from the crest of the wall at an angle of 20° with the horizontal. Assuming $\phi = 30°$ and $\delta = 20°$, determine the total active earth pressure using the analytical approach of Coulomb. (S.V.U.—U.Tech. (Part-time)—Sep., 1982)

$H = 3.6$ m $\phi = 20°$ $\beta = 20°$ $\alpha = 81°$ $\delta = 12°$ $\psi = \alpha - \delta = 69°$

Since $\beta = \phi$, the special case of Rebhann's construction for this condition is applied. The triangle *CGL* is constructed from any arbitrary point *C*.

$$P_a = \frac{1}{2}\gamma x^2.\sin\psi = \frac{1}{2}\times 18.9\times(3.85)^2\sin 69° = \mathbf{131\ kN/m.\ run}$$

The rupture surface cannot be located in this case.

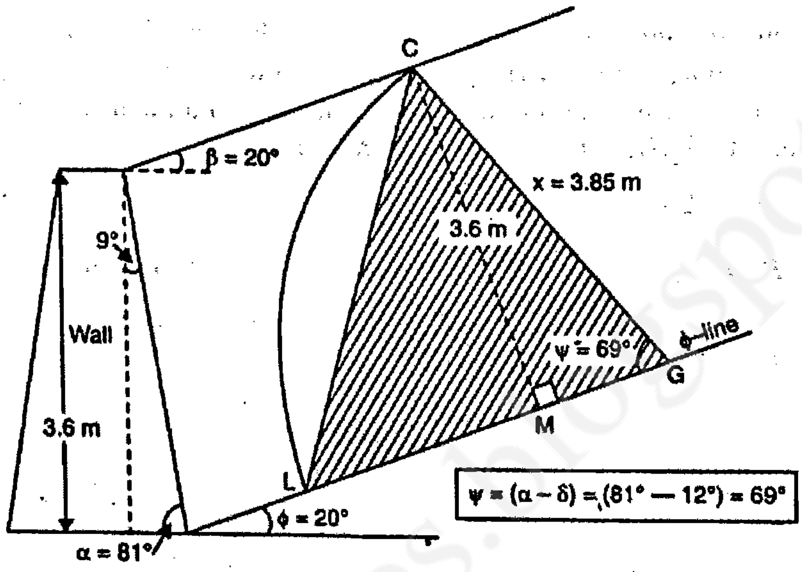


Fig. 13.66 Special case of Rebhann's construction when $\beta = \phi$ (Ex. 13.21)

Example 13.22: A masonry wall with vertical back has a backfill 5 m behind it. The ground level is horizontal at the top and the ground water table is at ground level. Calculated the horizontal pressure on the wall using Coulomb's earth pressure theory. Assume the unit weight of saturated soil is 15.3 kN/m³. Cohesion = 0. $\phi = 30°$. Friction between wall and earth = 20°.

(S.V.U.—B.E., (N.R.)—Sep., 1968)

$H = 5$ m $c = 0$ $\phi = 30°$ $\delta = 20°$ $\gamma_{sat} = 15.3$ kN/m³

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha.\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

since $\alpha = 90°$ and $\beta = 0°$ in this case,

$$K_a = \frac{\cos^2\phi}{\cos\delta\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin\phi}{\cos\delta}}\right]^2}$$

$$= \frac{\cos^2 30°}{\cos 20°\left[1+\sqrt{\frac{\sin 50°.\sin 30°}{\cos 20°}}\right]^2} = 0.132$$

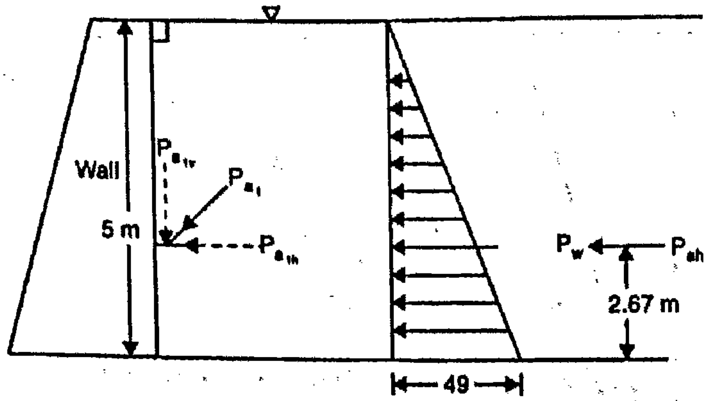


Fig. 13.67 Horizontal thrust on wall from submerged fill from Coulomb's theory (Ex. 13.22)

$$P_{a_1} = \frac{1}{2}\gamma H^2.K_a = \frac{1}{2}\times(15.30 - 9.8)\times 5^2 \times 0.132 \text{ kN/m} = 9.06 \text{ kN/m}$$

(since $\gamma' = \gamma_{sat} - \gamma_w$)

This is inclined at 20° with the horizontal.

∴ Its horizontal component, $P_{a_{1h}}$ = 9.06 × cos 20° = 8.50 kN/m

Water pressure, $P_w = \frac{1}{2}\gamma_w H^2 = \frac{1}{2}\times 9.81\times 5^2 \text{kN/m}$ = 122.6 kN/m

Total horizontal thrust, $P_{ah} = P_{a_{1h}} + P_w$ = 8.5 + 122.6 = 131.1 kN/m.

This will act at (1/3) H or 2.67 m above the base of the wall, as shown in Fig. 13.67.

Example 13.23: A retaining wall 4.5 m high with a vertical back supports a horizontal fill weighing 18.60 kN/m³ and having ϕ = 32°, δ = 20°, and c = 0. Determine the total active thrust on the wall by Culmann's method. (S.V.U.—B.E. (R.R.).—Sep., 1978)

γ = 18.6 kN/m³ ϕ = 32° c = 0 δ = 20° for the fill

Active thrust, $P_a = FF'$

= 51.5 kN/m. run

Check:

$$K_a \text{ from Coulomb's formula} = \frac{\cos^2 32°}{\cos 20°\left[1+\sqrt{\frac{\sin 52°.\sin 32°}{\cos 20°}}\right]^2}$$

= 0.2755

$$P_a = \frac{1}{2}\gamma H^2 K_a = \frac{1}{2}\times 18.6\times 4^2\times 5\times 0.2755 = 51.9 \text{ kN/m}$$

The Culmann value agrees excellently with this value.

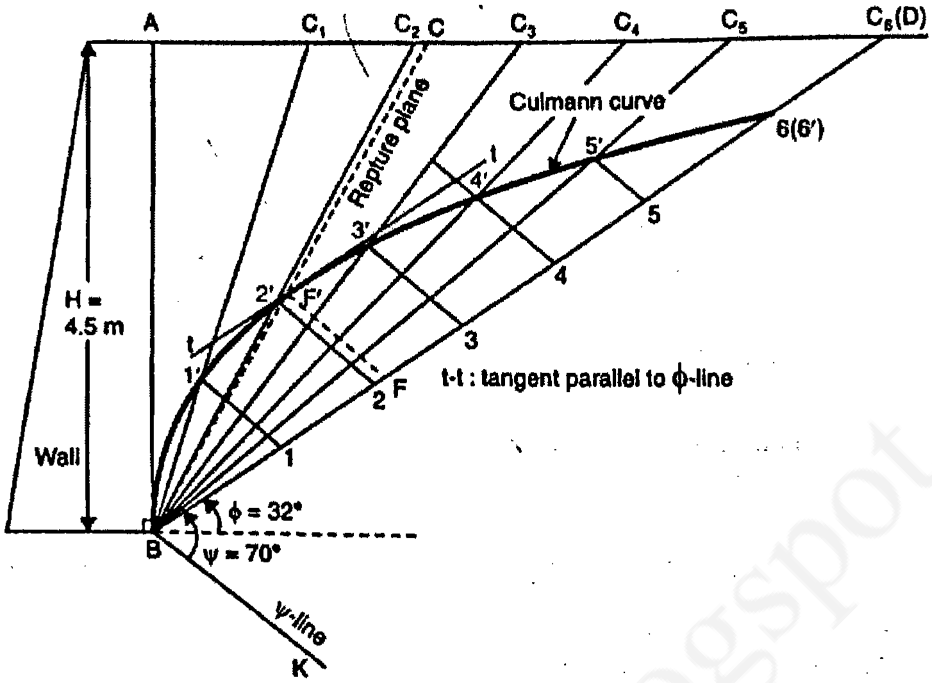


Fig. 13.68 Culmann's method (Ex. 13.23)

Example 13.24: **A retaining wall with its face inclined at 75° with horizontal is 10 m high and retains soil inclined at a uniform surcharge angle of 10°. If the angle of internal friction of the soil is 36°, wall friction angle 18°, unit weight of soil 15 kN/m^3, and a line load of intensity 90 kN per metre run of the wall acts at a horizontal distance of 5 m from the crest, determine the active thrust on the wall by Culmann's method.**

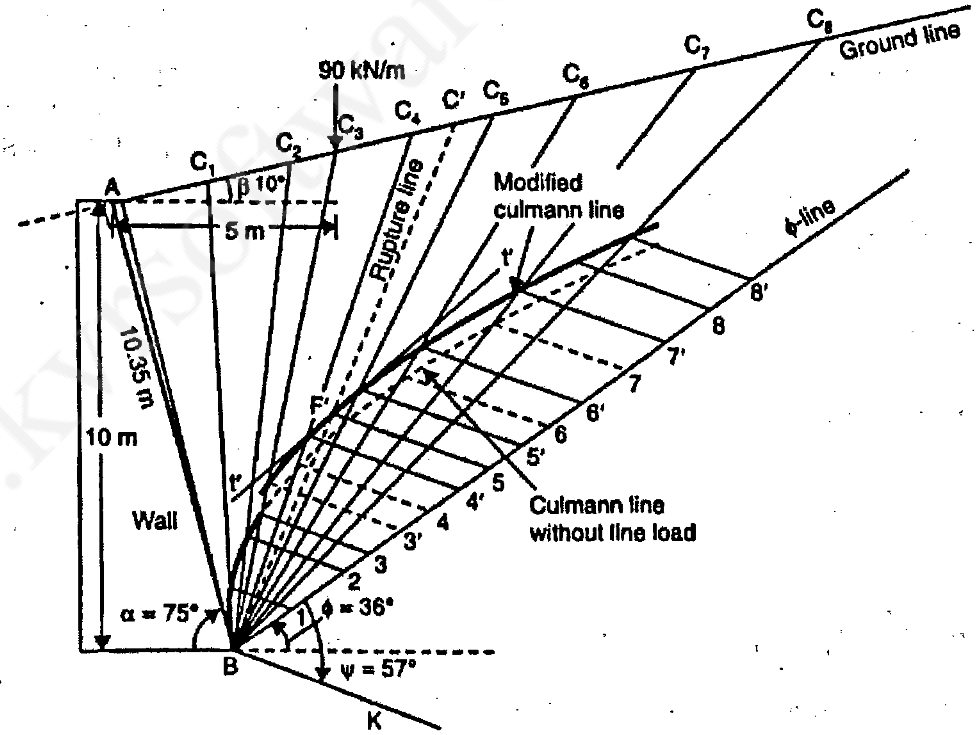


Fig. 13.69 Culmann's method for line load on backfill (Ex. 13.24)

$t'\,t'$: Tangent to the modified Culmann line, parallel to ϕ-line.

$$\alpha = 75° \quad \phi = 36° \quad \delta = 18° \quad \beta = 10° \quad \gamma = 15 \text{ kN/m}^3$$

Active thrust, P_a, on the wall per metre run = Vector $F'G'$ = 360 kN.

Example 13.25: A retaining wall 4.5 m high with vertical back supports a backfill with horizontal surface. The unit weight of the fill is 18 kN/m³ and the angle of internal friction is 36°. The angle of wall friction may be taken as 18°. A footing running parallel to the retaining wall and carrying a load of 18 kN/m is to be constructed. Find the safe distance of the footing from the face of the wall so that there is no increase in lateral pressure on the wall due to the load of the footing. *tt*: tangent to the Culmann-curve without line load, parallel to the ϕ-line.

The safe distance beyond which the line load does not increase the lateral pressure is 3.5 m in this case.

The Culmann-curve without the line load is drawn as usual. Now the modified Culmann-curve, with the line load included at C_1, C_2, C_3 ..., the borders of each of the wedges such as ABC_1, ABC_2, ABC_3, ..., is drawn. A tangent *tt* to the Culmann-curve without the line load is drawn parallel to ϕ-line to meet the modified curve with line load in F'. BF' is joined and produced to meet the surface in C, which gives the critical position of the line load, beyond which location, it does not affect the lateral pressure.

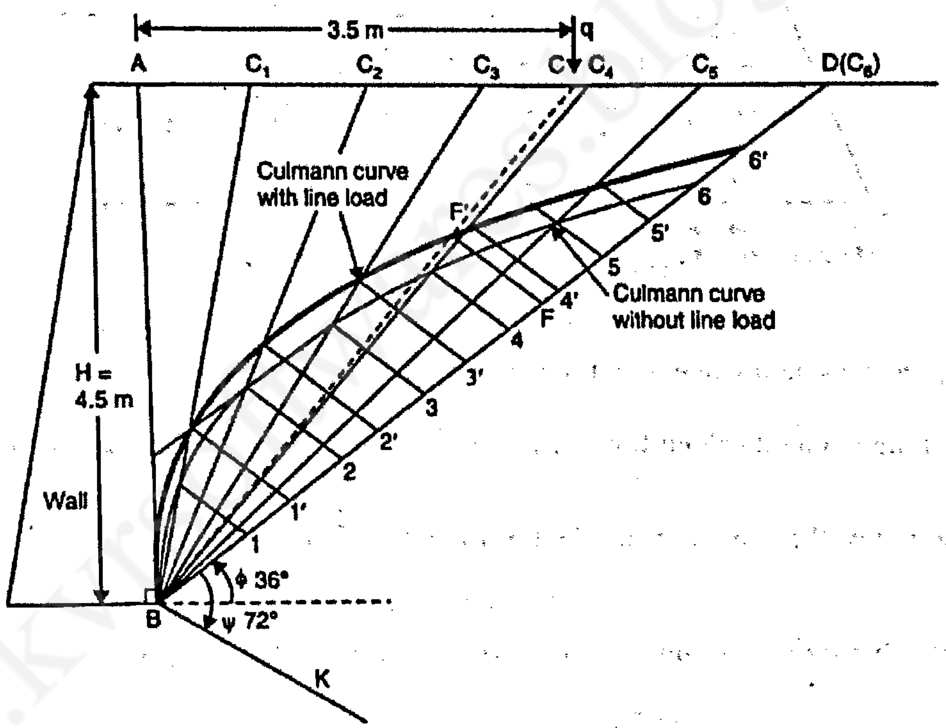


Fig. 13.70 Location of critical position of line load Culmann's method (Ex. 13.25)

Example 13.26: A masonry retaining wall is 1.5 m wide at the top, 3.5 m wide at the base and 6 m high. It is trapezoidal in section and has a vertical face on the earth side. The backfill is level with top. The unit weight of the fill is 16 kN/m³ for the top 3 m and 23 kN/m³ for the rest of the depth. The unit weight of masonary is 23 kN/m³. Determine the total lateral pressure on the wall per metre run and the maximum and minimum pressure intensities of normal pressure at the base. Assume ϕ = 30° for both grades of soil. (S.V.U.—Four-year B.Tech.—July, 1984)

$$\phi = 30°\ K_a = \frac{1-\sin 30°}{1+\sin 30°} = 1/3$$

Horizontal pressure of soil at 3 m depth $= K_a\gamma_1 H_1 = \frac{1}{3}\times 16\times 3 = 16$ kN/m^2

Lateral earth pressure at 6 m depth $= K_a(\gamma_1 H_1 + \gamma_2 H_2)$

$$= \frac{1}{3}(16\times 3 + 18\times 3) = 34 \text{ kN/m}^2$$

Total active thrust per metre run of the wall,

$$P_a = \frac{1}{2}\times 3\times 16 + 3\times 16 + \frac{1}{2}\times 3\times 18 = 24 + 48 + 27 = 99 \text{ kN}$$

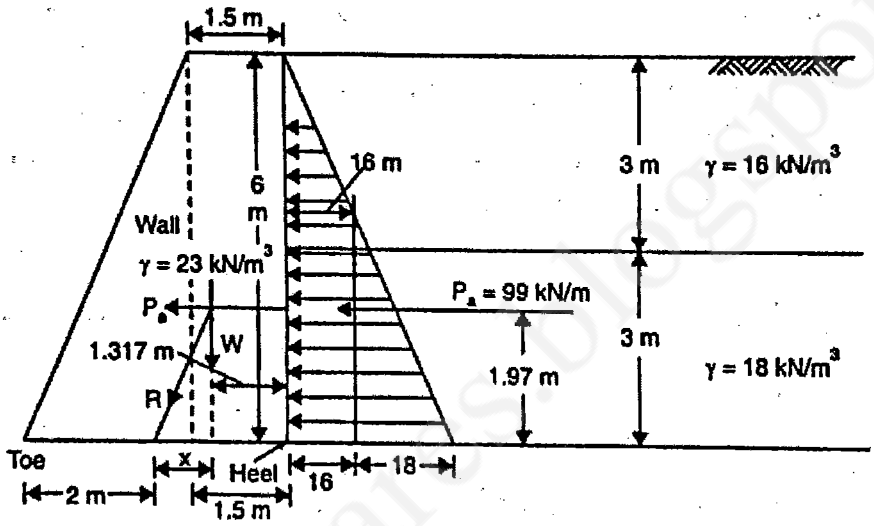


Fig. 13.71 Retaining wall (Ex. 13.26)

Let $\bar{z}$ metres be the height of the point of action above its base.

By taking moments about the base, $\bar{z} = \frac{(24\times 4 + 48\times 1.5 + 27\times 1)}{99} = 1.97$ m

Weight of wall per metre run, $W = 6\times 1.5\times 23 + \frac{1}{2}\times 2\times 6\times 23$

$$= 207 + 138 = 345 \text{ kN}$$

Let the distance of its point of action from the vertical face be $\bar{x}$ m.

By moments, $\bar{x} = \frac{\left(207\times 0.75 + 138\times \frac{13}{6}\right)}{345} = 1.317$ m

Let x metres be the distance from the line of action of W to the point where the resultant strikes the base.

$$\frac{x}{1.97} = \frac{P_a}{W} \quad \therefore \quad x = \frac{1.97\times 99}{345} = 0.565 \text{ m}$$

Eccentricity, $e = (1.317 + 0.565 - 1.750) = 0.132$ m

Since this is less than (1/6) b or (1/6) × 3.5 m, no tension occurs at the base.

Vertical pressure intensity at the base, $\sigma = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right) = \frac{345}{3.5}\left(1 \pm \frac{6 \times 0.132}{3.5}\right)$

or $\sigma_{max} = 120.88$ kN/m² at the toe

and $\sigma_{min} = 76.26$ kN/m, at the heel.

Example 13.27: A trapezoidal masonry retaining wall 1 m wide at top and 3 m wide at its bottom is 4 m high. The vertical face is retaining soil ($\phi = 30°$) at a surcharge angle of 20° with the horizontal. Determine the maximum and minimum intensities of pressure at the base of the retaining wall. Unit weights of soil and masonary are 20 kN/m³ and 24 kN/m³ respectively. Assuming the coefficient of friction at the base of the wall as 0.45, determine the factor of safety against sliding. Also determine the factor of safety against overturning.

(S.V.U.—B.E., (Part-time)—Dec.,1981)

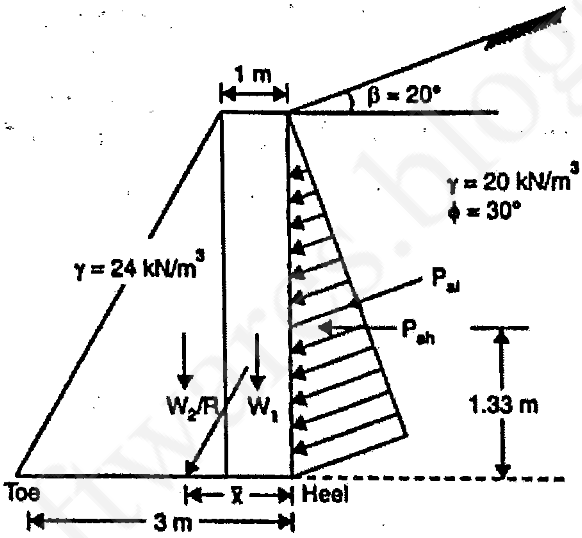


Fig. 13.72 Retaining wall (Ex. 13.27)

For backfill,

$$\gamma = 20 \text{ kN/m}^3 \quad \phi = 30° \quad \beta = 20°$$

$$K_{ai} = \cos\beta . \frac{(\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi})}{(\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi})}$$

$$= \cos 20° . \frac{(\cos 20° - \sqrt{\cos^2 20° - \cos^2 30°})}{(\cos 20° + \sqrt{\cos^2 20° - \cos^2 30°})}$$

$$= 0.414$$

$$P_{ai} = \frac{1}{2}\gamma H^2 . K_{ai} = \frac{1}{2} \times 20 \times 4^2 \times 0.414 = 66.24 \text{ kN/m}$$

This acts at 1.33 m above the base, at an angle of 20° with the horizontal.

$$P_{ah} = P_{ai} \cos \beta = 66.24 \cos 20° = 62.25 \text{ kN/m}$$

$$P_{av} = P_{ai} \sin \beta = 66.24 \sin 20° = 22.66 \text{ kN/m}$$

W_1, wt of the rectangular portion of the wall = 1 × 4 × 24 = 96 kN

W_2, wt of the triangular portion of the wall = $\frac{1}{2} \times 2 \times 4 \times 24$ = 96 kN

W_1 acts at 0.50 m and W_2 at 1.67 m from the vertical face.

$$\Sigma V = W_1 + W_2 + P_{av} = 96 + 96 + 22.66 = 214.66 \text{ kN}$$

The distance of the point where the resultant strikes the base from the heel,

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{(96 \times 0.50 + 96 \times 1.67 + 62.25 \times 1.33)}{214.66} = 1.357 \text{ m}$$

$$e = \frac{b}{2} - \bar{x} = 1.500 - 1.357 = 0.143 \text{ m}$$

$$\sigma_{max}, \text{ at the heel} = \frac{\Sigma V}{b}\left(1 + \frac{6e}{b}\right) = \frac{214.66}{3}\left(1 + \frac{6 \times 0.143}{3}\right) = 92 \text{ kN/m}^2$$

$$\sigma_{min}, \text{ at the toe} = \frac{\Sigma V}{b}\left(1 - \frac{6e}{b}\right) = \frac{214.66}{3}\left(1 - \frac{6 \times 0.143}{3}\right) = 51 \text{ kN/m}^2$$

These are intensities of normal pressures at the base.

Check for sliding:

Factor of safety against sliding, $\eta_s = \frac{\mu . N}{T}$

$$= \frac{0.45 \times 214.66}{162.25} = 1.55$$

This is O.K.

Check for overturning:

Factor of safety against overturning, η_0

$$= \frac{\text{Restoring moment about the toe}}{\text{Overturning moment about the toe}}$$

$$= \frac{(96 \times 2.5 + 96 \times 1.33 + 22.66 \times 3)}{62.25 \times 1.33}$$

$$= 5.25$$

This is excellent.

SUMMARY OF MAIN POINTS

1. The property of soil by virtue of which it exerts lateral pressure influences the design of earth-retaining structures, the most common of them being a retaining wall.

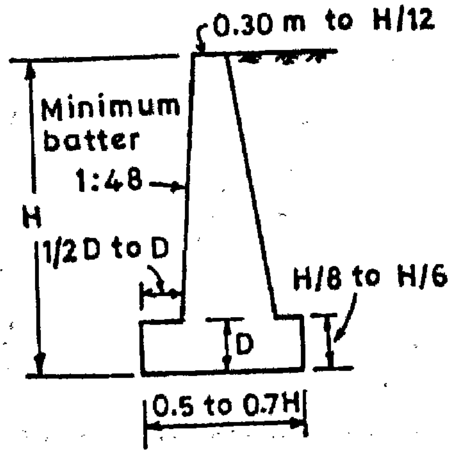


Fig. 12.30 Tentative dimensions for a gravity retaining wall

4. Width of stem at top: Min. of 200 mm but preferably 300 mm for case of casting.
5. Batter of front face: 1 hor. to 48 ver. (min.) to 1: 16.
   The gravity walls are proportioned such that the resultant falls within the middle one-third of the base. The trial dimensions are shown in Fig. 12.30.

EXAMPLES

Example 12.1 A retaining wall with a smooth vertical back retains sand backfill for a depth of 6m. The backfill has a horizontal surface and has the following properties:

$$c' = 0\,,\ \phi' = 28°\ ;\ \gamma = 16\ \text{kN/m}^3\ ;\ \gamma_{sat} = 20\ \text{kN/m}^3$$

Calculate the magnitude of the total thrust against the wall for the conditions given below:

(a) Backfill fully drained but the top of the wall is restrained against yielding; (b) backfill fully drained and the wall is free to yield, and (c) wall free to yield, water table at 3 m depth and there is no drainage. Determine the point of application of the resultant thrust for case (c).

Solution:

(a) If the wall is restrained against yielding, the lateral pressure that would develop against the wall would be earth pressure 'at rest'.

Coefficient of earth pressure at rest K_0 can be calculated from the equation $K_0 = 1 - \sin\phi'$

Hence $$K_0 = 1 - \sin 28° = 0.53$$

For this case, $$\gamma = 16\ \text{kN/m}^3.$$

Fig. 12.31(a) shows the pressure distribution diagram for this case.

Lateral pressure, $$p_0 = K_0\,\gamma\,z$$

At $$z = 6\ \text{m},\ \ p_0 = 0.53 \times 16 \times 6 = 50.88\ \text{kN/m}^2$$

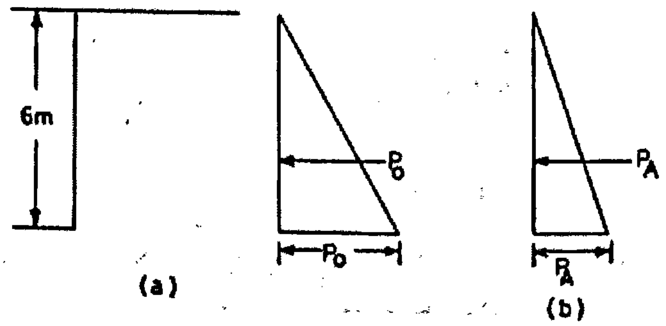


Fig. 12.31 Pressure distribution diagram — Example 12.1

Total thrust per meter length of the wall is

$$P_0 = \frac{1}{2} \times 50.88 \times 6 = 152.64 \text{ kN}$$

(b) For this case again, $\gamma = 16$ kN/m^3, but the lateral pressure is the active earth pressure.

$$\phi' = 28° ; K_A = \frac{1 - \sin\phi}{1 + \sin\phi} = 0.36$$

$$p_A = K_A \gamma z$$

at $z = 6$ m, $\quad p_A = 0.36 \times 16 \times 6 = 34.56$ kN/m^2

and $\quad P_A = \frac{1}{2} \times 34.56 \times 6 = 103.68$ kN/m length

Fig. 12.31(b) shows the pressure distribution.

(c) Fig. 12.32 shows the pressure distribution diagram. Above the water table, $\gamma = 16$ kN/m^3 and below the water table, $\gamma = \gamma' = \gamma_{sat} - \gamma_w = 20 - 9.8 = 10.2$ kN/m^3.
The lateral thrust due to water is shown as P_3.

$$p_1 = K_A \gamma H_1 = 0.36 \times 16 \times 3 = 17.28 \text{ kN/m}^2$$

$$p_2 = K_A \gamma' H_2 = 0.36 \times 10.2 \times 3 = 11 \text{ kN/m}^2$$

$$p_3 = \gamma_w H_2 = 9.8 \times 3 = 29.4 \text{ kN}$$

The total thrusts which are [illegible] in Fig. 12.32 are calculated as shown below:

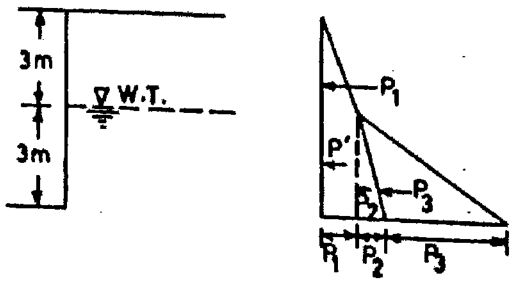


Fig. 12.32 Retaining wall and pressure distribution diagram—Example 12.1

$$P_1 = \frac{1}{2} \times 17.28 \times 3 = 25.92 \text{ kN acting at } (3 + \frac{1}{3} \times 3) = 4 \text{ m from base}$$

$$P' = 17.28 \times 3 = 51.84 \text{ kN acting at } \frac{3}{2} = 1.5 \text{ m from base}$$

$$P_2 = \frac{1}{2} \times 11 \times 3 = 16.5 \text{ kN acting at } \frac{1}{3} \times 3 = 1 \text{ m from base}$$

$$P_3 = \frac{1}{2} \times 29.4 \times 3 = 44.1 \text{ kN acting at } \frac{1}{3} \times 3 = 1 \text{ m from base}$$

It can be seen that the lateral thrust due to water contributes substantially to the total lateral thrust.

$$\text{Total thrust } P = P_1 + P' + P_2 + P_3$$

$$= 25.92 + 51.84 + 16.5 + 44.1$$

$$= 138.36 \text{ kN/m length}$$

The distance of the resultant P from the base of the wall can obtained by taking moments about the base.

$$\overline{H} = \frac{25.92 \times 4 + 51.84 \times 1.5 + 16.5 \times 1 + 44.1 \times 1}{138.36} = \frac{242}{138.36} = 1.75 \text{ m}$$

Example 12.2 What is the increase in total pressure in Ex. 12.1, case (c), if the backfill supports a uniformly distributed load of 25 kN/m^2?

Solution:

If a uniform surcharge load q per unit area is applied over the backfill surface, the increase in active earth pressure Δp_A at every point on the back of the wall is given by:

$$\Delta p_A = K_A q$$

$$= 0.36 \times 25 = 9 \text{ kN/m}^2 \text{ (Fig. 12.33)}$$

Hence the increase in total lateral pressure $\Delta P_A = 9 \times 6 = 54$ kN/m length.

The point of application of the thrust is at a distance $\frac{1}{2} \times 6 = 3$ m from the base.

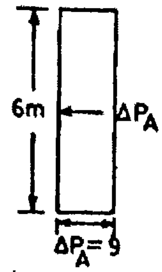


Fig. 12.33 Earth pressure due to surcharge — Example 12.2

Example 12.3 A retaining wall with a smooth vertical back is 10 m high and retains a two layer sand backfill with the following properties:

$$0 - 5 \text{ m depth: } c' = 0,\ \phi' = 30°,\ \gamma = 18 \text{ kN/m}^3$$

$$\text{Below 5 m}: c' = 0,\ \phi' = 34°,\ \gamma = 20 \text{ kN/m}^3$$

Show the active earth pressure distribution, assuming that the water table is well below the base of the wall.

Solution:

When the backfill consists of more than one soil layer, the lateral pressure distribution for each of the layers is worked out and a combined diagram drawn. At the interface of two layers, there will be a break in the pressure distribution diagram, since there will be two values of pressure – one value at the base of the upper layer and another at the top of the lower layer. For the lower layer, the upper layer will act as a surcharge.

$$K_{A_1} \text{ for the upper layer } = \frac{1 - \sin 30°}{1 + \sin 30°} = 0.333$$

K_{A_2} for the lower layer $= \dfrac{1-\sin 34°}{1+\sin 34°} = 0.283$

Active pressure distribution for the upper layer:

$$z = 0 \text{ m; vertical pressure } p_V = 0\ ;\ p_A = 0$$

$$z = 5 \text{ m; vertical pressure } p_V = 18 \times 5 = 90 \text{ kN/m}^2$$

$$p_A = K_{A_1} p_V = 0.333 \times 90 = 30 \text{ kN/m}^2$$

Active pressure distribution for the lower layer:

$$z = 5 \text{ m; vertical pressure } p_V = 90 \text{ kN/m}^2$$

$$p_A = K_{A_2} p_V = 0.283 \times 90 = 25.47 \text{ kN/m}^2$$

$$z = 10 \text{ m; } p_V = 90 + 20 \times 5 = 190 \text{ kN/m}^2$$

$$p_A = 0.283 \times 190 = 53.77 \text{ kN/m}^2$$

The active pressure distribution is shown in Fig. 12.34.

In reality, there cannot be a sudden change in lateral pressure since shear stresses which develop along the interface have not been considered. But this does not introduce any serious error in the magnitude and direction of the resultant thrust.

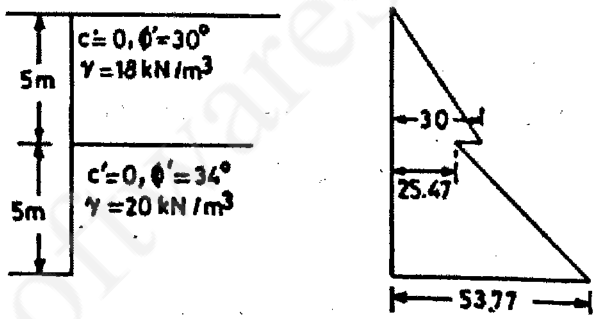


Fig. 12.34 Retaining wall and earth pressure diagram—Example 12.3

Example 12.4 A retaining wall, 8 m high, with a smooth vertical back, retains a clay backfill with $c' = 15 \text{ kN/m}^2$, $\phi' = 15°$ and $\gamma = 18 \text{ kN/m}^3$. Calculate the total active thrust on the wall assuming that tension cracks may develop to the full theoretical depth.

Solution:

The active pressure at a depth z in a $c-\phi$ soil is given by:

$$p_A = K_A p_V - 2c\sqrt{K_A}$$

$$K_A = \frac{1-\sin\phi}{1+\sin\phi} = 0.588$$

At $z = 0,\ p_V = 0\ ;\ p_A = -2 \times 15\sqrt{0.588} = -23 \text{ kN/m}^2$

At $z = 8 \text{ m},\ p_V = 18 \times 8 \text{ kN/m}^2\ ;\ p_A = 0.588 \times 18 \times 8 - 23 = 61.67 \text{ kN/m}^2$

$$p_A = 0 \text{ at } z_0 = \frac{2c}{\gamma\sqrt{K_A}} = \frac{2\times 15}{18\sqrt{0.588}} = 2.17 \text{ m}$$

From Fig. 12.35, it can be seen that the lateral pressure is negative or tensile upto a depth of 2.17 m. The soil tends to break away from the wall and tension cracks develop in the soil. Hence, the resultant active thrust is obtained by determining the area of the hatched portion of the pressure diagram.

$$\therefore P_A = \frac{1}{2}\times 61.67\times 5.83 = 179.8 \text{ kN/m}$$

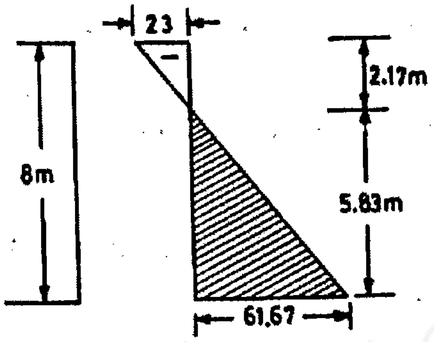


Fig. 12.35 Earth pressure diagram—Example 12.4

Example 12.5 An excavation was made in saturated, soft clay ($\phi_u = 0$), with its sides more or less vertical. When the depth of excavation reached 6 m, the sides caved in. What was the approximate value of cohesion of the clay soil? Take unit weight of clay=20 kN/m^3.

Solution:

The depth of tension crack z_0 in a $c-\phi$ soil is equal to $\frac{2c}{\gamma\sqrt{K_A}}$ and the depth for which the total net pressure is zero is equal to 2 z_0 or $\frac{4c}{\gamma\sqrt{K_A}}$. An excavation in such a soil should be able to sustain its vertical faces upto a depth of $\frac{4c}{\gamma\sqrt{K_A}}$ without any lateral support.

Critical depth of cut in a $\phi_u = 0$ soil is, therefore, equal to $\frac{4c_u}{\gamma}$ since $K_A = 1$ for $\phi_u = 0$.

$$\therefore \quad \frac{4c_u}{4} = 6$$

or $$c_u = \frac{6\times 20}{4} = 30 \text{ kN/m}^2$$

Example 12.6 Fig. 12.36(a) shows a 3-layered backfill behind a 15 m high retaining wall with a smooth vertical back. Draw the active earth pressure distribution.

Solution:

Layer 1: ($c = 0$ soil) $$K_A = \frac{1-\sin 35^\circ}{1+\sin 35^\circ} = 0.27$$

$z = 0$; vertical pressure $p_V = 0$

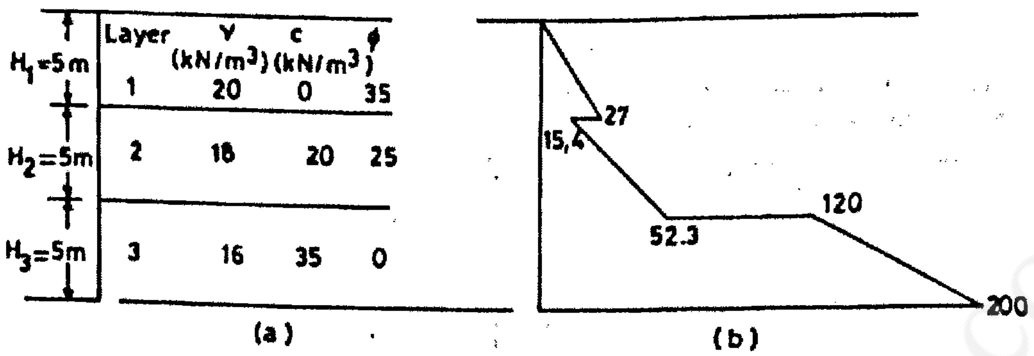


Fig. 12.36 (a) Retaining wall with backfill (b) Earth pressure diagram—Example 12.6

$$p_A = K_A p_V = 0$$

$$z = 5 \text{ m}; p_V = 20 \times 5 = 100 \text{ kN/m}^2$$

$$p_A = 100 \times 0.27 = 27 \text{ kN/m}^2$$

Layer 2: ($c - \phi$ soil)

$$K_A = \frac{1 - \sin 25°}{1 + \sin 25°} = 0.41$$

$$z = 5 \text{ m}; p_V = 20 \times 5 = 100 \text{ kN/m}^2$$

$$p_A = K_A p_V - 2c\sqrt{K_A}$$

$$P_A = 0.41 \times 100 - 2 \times 20\sqrt{0.41} = 15.4 \text{ kN/m}^2$$

$$z = 10 \text{ m}; p_V = 100 + 18 \times 5 = 190 \text{ kN/m}^2$$

$$p_A = 0.41 \times 190 - 2 \times 20\sqrt{0.41} = 52.3 \text{ kN/m}^2$$

Layer 3:

$$K_A = 1\ (\phi_u = 0 \text{ soil})$$

$$z = 10 \text{ m}; p_V = 190 \text{ kN/m}^2$$

$$p_A = p_V - 2c_u = 190 - 2 \times 35 = 120 \text{ kN/m}^2$$

$$z = 15 \text{ m}; p_V = 190 + 16 \times 5 = 270 \text{ kN/m}^2$$

$$p_A = 270 - 2 \times 35 = 200 \text{ kN/m}^2$$

Figure 12.36(b) shows the active pressure distribution (not to scale)

Example 12.7 A retaining wall 6 m high, with a smooth vertical back is pushed against a soil mass having $c' = 40$ kN/m^2 and $\phi' = 15°$; $\gamma = 19$ kN/m^3. What is the total Rankine passive pressure, if the horizontal soil surface carries a uniform load of 50 kN/m^2? What is the point of application of the resultant thrust?

Solution :

The passive pressure at a depth z in a $c - \phi$ soil is given by:

$$p_P = K_P p_V + 2c\sqrt{K_P}$$

where p_V is the vertical pressure at that depth

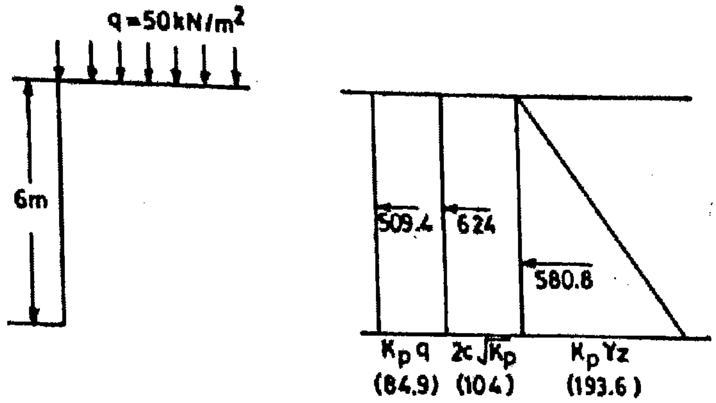


Fig. 12.37 Retaining wall and earth pressure diagram—Example 12.7

For the case of a uniform surcharge q acting over the backfill surface, $p_V = \gamma z + q$

$$p_P = K_P \gamma z + K_P q + 2c\sqrt{K_P}$$

$$K_P = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 15^\circ}{1-\sin 15^\circ} = 1.698$$

At $z = 0$,

$$p_P = 1.698 \times 50 + 2 \times 40 \times 1.3 = 84.9 + 104 = 188.9 \text{ kN/m}^2$$

At $z = 6$ m,

$$p_P = 1.698 \times 19 \times 6 + 84.9 + 104 = 193.6 + 188.9 = 382.5 \text{ kN/m}^2$$

The passive pressure distribution is shown in Fig. 12.37. The contributions of surcharge, cohesion and weight of the soil to the passive pressure are indicated separately on the diagram. They have been calculated by considering the area of the respective pressure distribution.

The total passive earth pressure

$$P_P = 509.4 + 624 + 580.8 = 1714.2 \text{ kN/m}$$

The distance of the point of application of the resultant passive thrust, H from the base can be determined by taking moments about the base.

$$H = \frac{6 \times 84.9 \times 3 + 6 \times 104 \times 3 + \frac{1}{2} \times 193.6 \times 6 \times \frac{6}{3}}{1714.2}$$

$$= \frac{1528.2 + 1872 + 1161.6}{1714.2}$$

$$= 2.66 \text{ m}$$

Example 12.8 The retaining wall shown in Fig. 12.38 retains a soil with the following properties:

$$c' = 0,\ \phi' = 34^\circ,\ \gamma = 19 \text{ kN/m}^3,\ \delta = 20^\circ$$

The backfill surface is sloping at an angle of 20° to the horizontal.

(a) Determine the total active thrust by Culmann's graphical construction.

(b) A vertical line load of 60 kN/linear meter is acting at a horizontal distance of 3.5 m from the wall parallel to the crest of the wall. What is the magnitude of total active thrust?

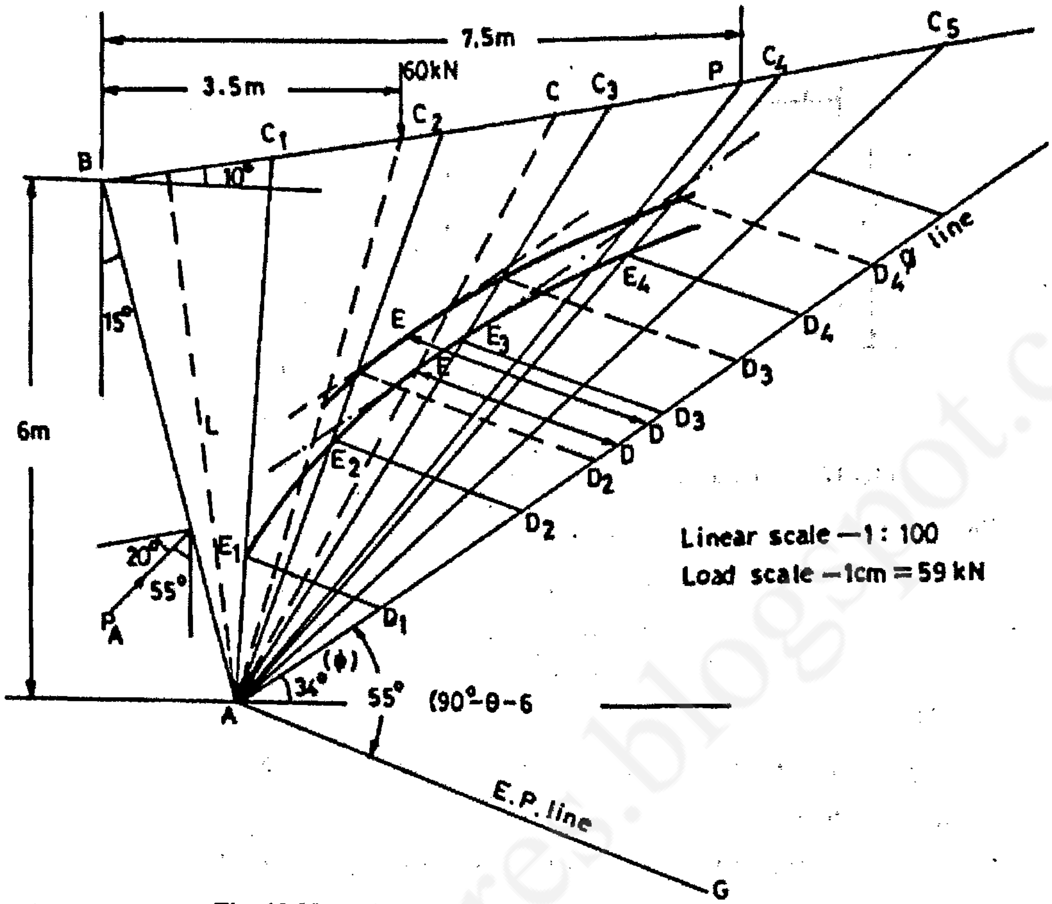


Fig. 12.38 Culmann's graphical construction—Example 12.8

(c) What is the minimum horizontal distance at which the line load can be located from the back of the wall, if there is to be no increase in the total active thrust?

Solution:

(a) In Fig. 12.38, the section of the wall has been drawn to scale. Trial planes such as AC_1, AC_2 etc., have been selected to intersect the surface at points C_1, C_2 etc., such that $BC_1 = C_1C_2 = C_2C_3$ (= 2 m) and so on. When the backfill surface is planar, each triangular sliding wedge such as ABC_1, ABC_2 etc., has its base on the ground surface and a constant perpendicular distance L from the common vertex A. The area and, hence, the weight of each sliding wedge is, thus, proportional to its base length measured from B. The weight of a wedge can, therefore, be represented by its base length. In Fig. 12.38, the weight of wedge ABC_1 is represented by AD_1 ($= BC_1$), that of ABC_2 by AD_2 ($= BC_2$) so on. The Culmann curve is drawn, as explained in Section 12.7. The magnitude of the total active thrust corresponding to the critical plane AC is given by:

$$P_A = W\frac{DE}{AD} = \frac{1}{2}\gamma L\frac{BC \cdot DE}{AD} = \frac{1}{2}\gamma L \cdot DE \;(\because\; AD = BC)$$

From Fig. 12.38,

$$L = 6.2 \text{ m} \quad DE = 2.5 \text{ m}$$

$$\therefore\; P_A = \frac{1}{2} \times 19 \times 6.2 \times 2.5 = 147.25 \text{ kN/m length}$$

Note: The load scale works out to 1 cm = 59 kN

(b) When a line load is acting on the soil surface, the weight of any trial wedge that has the line load acting on it will be increased by the magnitude of the line load and shall accordingly be represented on the ϕ - line, as shown in Fig. 12.38. The new Culmann curve is now completed. From Fig. 12.38, $DE = 2.9$ cm. Therefore, the magnitude of the total active thrust is:

$$P_A = 2.9 \times 59 = 170.8 \text{ kN/m length}$$

(c) The tangent drawn to the old Culmann curve (when no line load was considered) is continued to intersect the new Culmann curve (when line load was considered). The line joining this point of intersection with point A is continued till it intersects the backfill surface at P. If the line load were to act at P or beyond P, the active thrust would be the same as for the case without any line load.

From Fig. 12.38, the horizontal distance of P from the back of the wall is equal to 7.5 m.

Example 12.9 In Ex. 12.8(a), if the retaining wall is to be designed to withstand the effect of an earthquake, what would be the magnitude of the dynamic component of active earth pressure? $\alpha_h = 0.1\ g$.

Show the point of application of $P_{A(sta)}$ and the dynamic component of the active earth pressure.

Solution:

$$\lambda = \tan^{-1} \frac{W_{\alpha} h}{g} = \tan^{-1} 0.1 = 5.7°$$

Hence, the modified slope line AF is drawn at an angle $(\phi - \lambda)° = (34 - 5.7)°$, i.e., 28.3° to the horizontal at A. The resultant of the weight of the wedge W, and $0.1\ W$ is plotted along AF, instead of only W. Rest of the construction is similar to the case of the static load. Fig. 12.39 shows the modified construction. The linear scale and the load scale are the same as the ones used in Fig. 12.38.

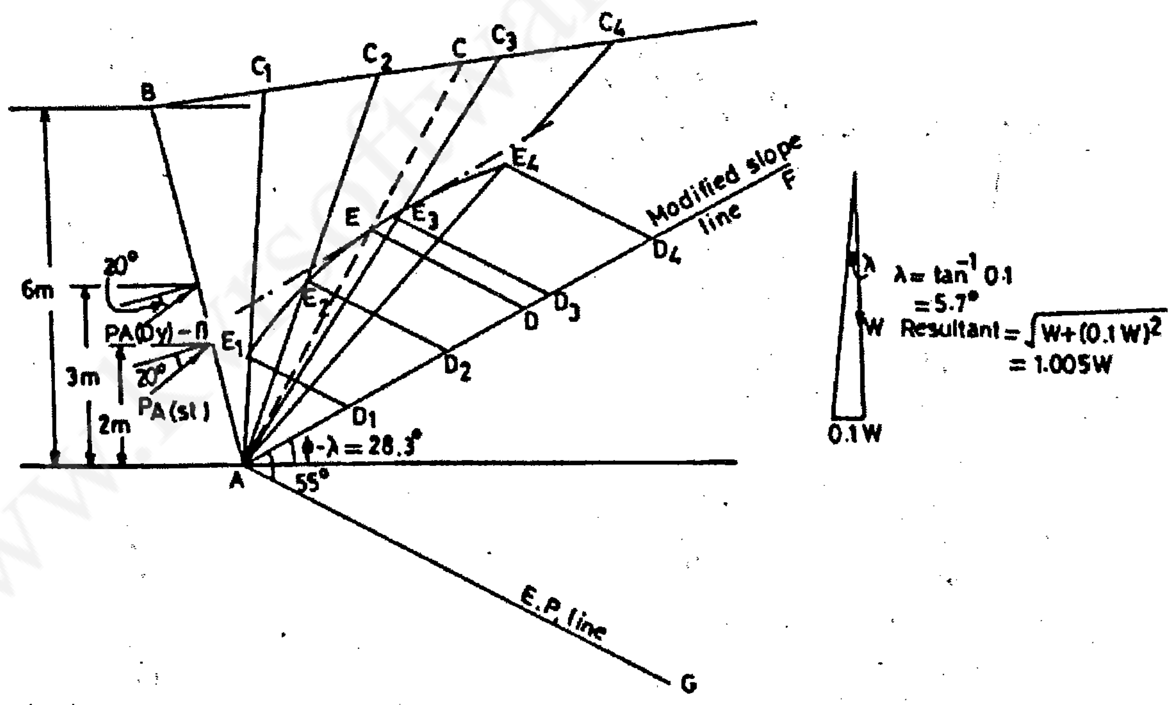


Fig. 12.39 Culmann's graphical construction—Example 12.9

Case-II: For Submerged Backfill

Net pressure in submerged backfill is due to the effect of

(i) Lateral earth pressure due to submerged soil

(ii) Hydrostatic pressure due to pore water

$\therefore \qquad p = p_E + p_w$

Where, p_E = Lateral earth pressure due to submerged soil = $k\gamma' z$

and p_w = hydrostatic pressure due to pore water = $\gamma_w z$

$\therefore \qquad p = k\gamma' z + \gamma_w z$ ...(iii)

At point A, $z = 0$

$p_1 = 0$

and $p_2 = 0$

At point B, $z = H$

$p_1 = k\gamma' H$

and $p_2 = p_2 = \gamma_w H$

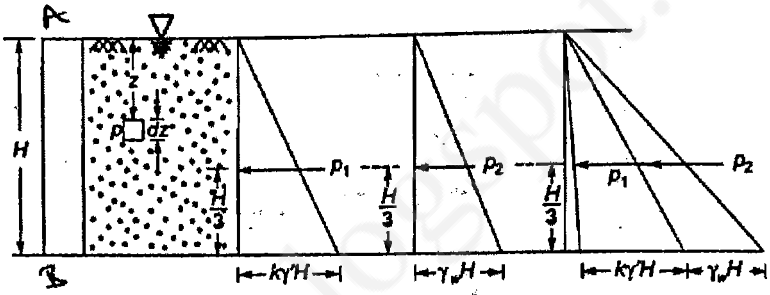


Fig. 11.9

Total lateral thrust,

$$P = P_E + P_w$$

$$= \frac{1}{2}k\gamma' H^2 + \frac{1}{2}\gamma_w H^2$$

The total thrust acts at a height of $\frac{H}{3}$ from the base.

Remember

- Never calculate total thrust due to submerged backfill as follows:

$$P = P_E + P_W$$

$$P = \frac{1}{2}k\gamma_{sat} H^2 + \frac{1}{2}\gamma_w H^2 \qquad \text{...(a)}$$

- We know, $\gamma_{sat} = \gamma' + \gamma_w$

$$\therefore \quad P = \frac{1}{2}k(\gamma' + \gamma_w)H^2 + \frac{1}{2}\gamma_w H^2$$

$$= \frac{1}{2}k\gamma' H^2 + \frac{1}{2}k\gamma_w H^2 + \frac{1}{2}\gamma_w H^2 \qquad \text{...(b)}$$

- In above relation term $\frac{1}{2}k\gamma_w H^2$ indicates that water follows earth pressure theory along with hydrostatic theory, which not true.

Example 11.1 A 15 m high rigid retaining wall with smooth vertical back retain a mass of moist cohesionless sand with horizontal surface and following properties:

$\gamma = 16$ kN/m^3 and $\phi = 32°$

(a) Compute the total lateral earth pressure at rest, and its location

(b) If subsequently the water table rises to the ground surface, determine the Increase the increase in earth pressure at rest. Take $k_0 = 0.45$.

Solution:

(a) Total earth pressure at rest,

$$P_0 = \frac{1}{2}k_0\gamma H^2 = \frac{1}{2}\times 0.45\times 16\times 15^2 = 810 \text{ kN/m}$$

Total earth pressure acts at

$$\frac{H}{3} = \frac{15}{3} = 5 \text{ m from base}$$

(b) When water table rises to ground surface, the soil becomes submerged

Now total lateral thrust,

$$P_2 = P_E + P_W$$
$$= \frac{1}{2}k_0\gamma' H^2 + \frac{1}{2}\gamma_w H^2$$
$$= \frac{1}{2}\times 0.45\times(14-9.8)\times 15^2 + \frac{1}{2}\times 9.8\times 15^2 = 1315.125 \text{ kN/m}$$

$\therefore$ Increase in earth pressure,

$$\Delta P = P_2 - P_0$$
$$= 1315.125 - 810.000 = 505.125 \text{ kN/m}$$

Example 11.2 A cohesionless soil with a void ratio of $e = 0.6$ and specific gravity of soil solids, $G_s = 2.65$ exists at a site where the water table is located at a depth of 2 m below the ground surface. Assuming a value of coefficient of earth pressure at rest $k_0 = 0.5$, calculate total lateral pressure at rest. Assume soil to be dry above the water table and saturated below the water table, use $\gamma_w = 9.81$.

Solution:

Dry unit weight of soil, $$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65\times 9.81}{1+0.6} = 16.25 \text{ kN/m}^3$$

Saturated unit weight of soil,

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.6)\times 9.81}{1+0.6} = 19.93 \text{ kN/m}^3$$

$\therefore$ Submerged unit weight of soil,

$$\gamma' = \gamma_{sat} - \gamma_w = 19.93 - 9.81 = 10.12 \text{ kN/m}^3$$

Lateral earth pressure at B (2 m from top)

$$p_B = k_0\gamma_d z = 0.5\times 16.25\times 2 = 16.25 \text{ kN/m}^2$$

Lateral earth pressure at C (5 m from top)

$$p_C = k_0\times\gamma_d\times 2 + k_0\times\gamma'\times 3$$
$$= 0.5\,(16.25\times 2 + 10.12\times 3) = 31.43 \text{ kN/m}^2$$

Hydrostatic pressure at C (5 m from top)

$$p_W = \gamma_w\times 3 = 9.81\times 3 = 29.43 \text{ kN/m}^2$$

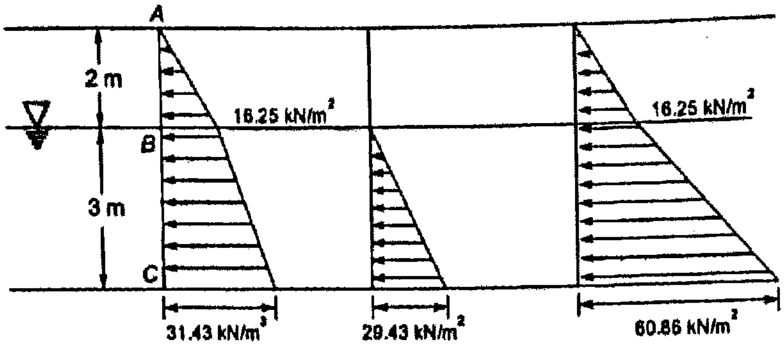


Fig. (a) Lateral earth pressure (b) Hydrostatic pressure (c) Total pressure

Total thrust, P = Area of lateral earth pressure dia. + Area of hydrostatic pressure dia.

$$= \left[\frac{1}{2}\times 16.25\times 2+\frac{1}{2}\times(16.25+31.43)\times 3\right]+\left[\frac{1}{2}\times 29.43\times 3\right]$$

$$= 87.770 + 44.145 = 131.915 \text{ kN/m}$$

11.5 Active and Passive Earth Pressure

11.5.1 Active Earth Pressure

In active state where wall moves away from the backfill, the portion of the backfill that is just behind the wall also moves along with it, leaving the rest of soil mass.

This portion of the backfill which moves along with the wall is termed as failure wedge.

The movement of the wedge is being resisted by the shear strength of the soil which develops over the rupture surface upward direction that causes the decrease of pressure acting on the wall.

This decrease in pressure with the movement of the wall takes place upto an extent when the entire shear strength of the soil is being mobilized and the pressure acting on the wall become minimum and is referred as active pressure.

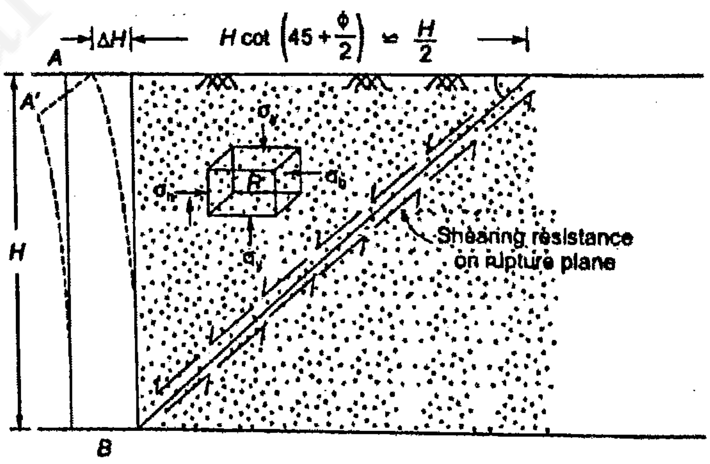


Fig. 11.10

For active state strain $\left(\frac{\Delta H}{H}\times 100\right)$ is

(i) 0.2% for dense soils

(ii) 0.5% for loose soils

11.5.2 Passive Earth Pressure

On the other hand, when wall moves towards the soil, a block of soil has tendency to move up and inside towards the fill. Hence shear resistance setup against the wall and resulting into increase in pressure at rest condition.

11.7 Various Cases of Earth Pressure in Cohesionless Soil

1. Dry or Moist Backfill

- Active earth pressure per unit length of wall

$$p_a = k_a \gamma z$$

At point A $(z = 0)$, $p_a = 0$

At point B $(z = H)$ $p_a = k_a \gamma H$

Total active thrust per unit length of wall

$$p_a = \frac{1}{2} \times (k_a \gamma z) \times H \times 1$$

$$= \frac{1}{2} k_a \gamma H^2$$

- Similarly, Passive earth pressure per unit length of wall

$$p_a = k_p \gamma z$$

At point A, $z = 0$ $p_p = 0$

At point B, $z = H$ $p_p = k_p \gamma H$

Total passive thrust per unit length of wall

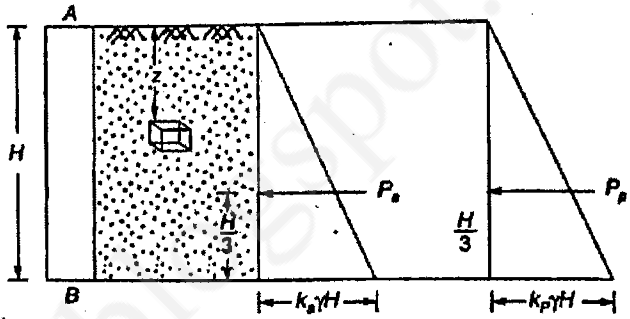


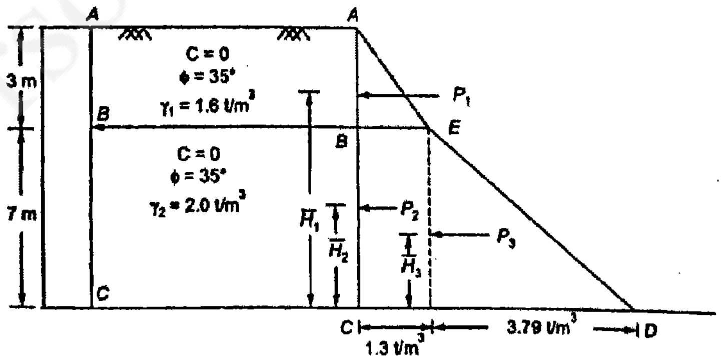
Fig. 11.16

$$p_a = \frac{1}{2} \times (k_p \gamma z) \times H \times 1 = \frac{1}{2} k_p \gamma H^2$$

- Line of action of total thrust (both p_a and p_p) pass through $\frac{H}{3}$ height from the base of wall.

Example 11.3 A retaining wall 10 m high retain a cohesionless soil with an angle of internal friction 35°. The surface is level with the top of wall. The unit weight of the top 3 m of the fill is 1.6 t/m³ and that of the rest is 2.0 t/m³. Find the magnitude and point of application of resultant active thrust.

Solution:



Coefficient of active earth pressure,

$$k_a = \frac{1-\sin\phi°}{1+\sin\phi°} = \frac{1-\sin 35°}{1+\sin 35°} = 0.271$$

Active pressure at B, 3 m below the top of wall

$$p_1 = k_a\gamma_1 H_1 = 0.271 \times 1.6 \times 3 = 1.3 \text{ t/m}^2$$

Active pressure at C, 10 m below the top of wall

$$p_2 = k_a\sigma_v = k_a(\gamma_1 \times 3 + \gamma_2 \times 7)$$
$$= 0.271 \times (1.6 \times 3 + 2.0 \times 7) = 1.3 + 3.79$$
$$= 5.09 \text{ t/m}^2$$

S.No.	Thrust	Point of Application of thrust ($\bar{H}$) from base of wall	$P_i \times \bar{H}_i$	$H = \frac{\Sigma P\bar{H}_i}{\Sigma P_i}$
1.	$P_1 = \frac{1}{2} \times 1.3 \times 3 \times 1 = 1.95$ t/m	$\bar{H}_1 = 7 + \frac{3}{3} = 8$ m	$= 1.95 \times 8 = 15.6$	$\bar{H} = \frac{78.39}{24.33} = 3.22$m
2.	$P_2 = 1.3 \times 7 \times 1 = 9.1$ t/m	$\bar{H}_2 = \frac{7}{2} = 3.5$ m	$= 9.1 \times 3.5 = 31.85$	
3.	$P_3 = \frac{1}{2} \times 3.79 \times 7 \times 1 = 13.265$ t/m	$\bar{H}_3 = \frac{7}{3} = 2.33$ m	$= 13.28 \times 2.33 = 30.94$	
	$\sum P_i = 24.33$		$\sum P_i\bar{H}_i = 78.39$	$\bar{H} = 3.22$ m

Hence, the resultant active thrust of 24.33 t/m acts on the wall at 3.22 m above its base.

2. Submerged Backfill

For submerged cohesionless backfill, the lateral active pressure at any depth can be split into two viz. one due to soil grain and other due to hydrostatic water.

$$p_a = p_{a,E} + p_w$$
$$= k_a\gamma' z + \gamma_w z$$

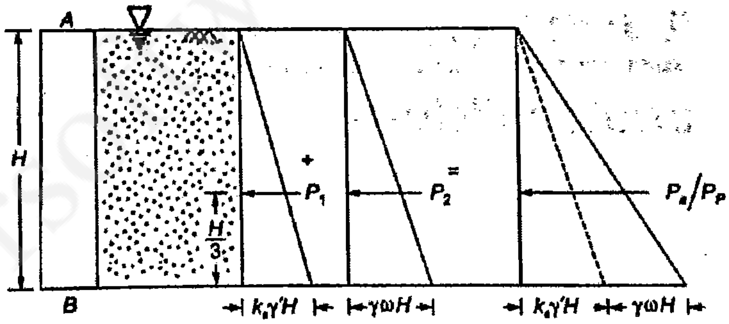


Fig. 11.17

- At point A, ($z = 0$), $p_1 = k_a\gamma' z = 0$, $p_2 = \gamma_w z = 0$

  At point B ($z = H$), $p_1 = k_a\gamma' H$, $p_2 = \gamma_w H$
- For total active thrust per unit length of wall,

$$P_1 = \frac{1}{2} \times k_a\gamma' H \times H \times 1 = \frac{1}{2}k_a\gamma' H^2$$

$$P_2 = \frac{1}{2} \times \gamma_w H \times H \times 1 = \frac{1}{2}\gamma_w H^2$$

$\therefore$ Total active thrust, $$P_a = P_1 + P_2 = \frac{1}{2}k_a\gamma' H^2 + \frac{1}{2}\gamma_w H^2$$

- Similarly, total passive thrust per unit length of wall

$$P_p = P_1 + P_2 = \frac{1}{2}k_P\gamma' H^2 + \frac{1}{2}\gamma_w H^2$$

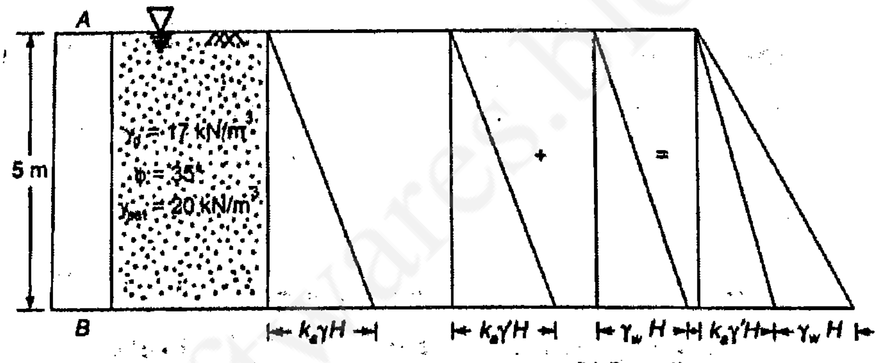
- Line of action of total thrust (both P_a and P_p) pass through $\frac{H}{3}$ height from the base of wall.

Example 11.4 A retaining vertical wall 5 m high, retaining a sand of unit weight 17 kN/m³ for which θ = 35°, the surface of the sand is horizontal and the water table is below the bottom of wall. Determine the percentage change in the active thrust on wall, if water table rises to ground level. The saturated unit weight of sand is 20 kN/m³.

Solution:

Coefficient of active pressure,

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 35°}{1+\sin 35°} = 0.271$$



(a) Case -I (b) Case -II

Case-I: Pressure at base of wall,

$$p_1 = k_a\sigma_v = k_a\gamma H$$
$$= 0.271 \times 17 \times 5 = 23.035 \text{ kN/m}^2$$

Thrust, $$P_1 = \frac{1}{2}k_a\gamma H \times H \times 1 = \frac{1}{2}k_a\gamma H^2$$

$$= \frac{1}{2} \times 0.271 \times 17 \times 5^2 = 57.58 \text{ kN/m}$$

Case-II:

$$p_2 = p_E + p_w$$
$$= k_a\gamma' H + \gamma_w H$$

and thrust, $$P_2 = \frac{1}{2} \times (k_a\gamma' H + \gamma_w H) \times H \times 1$$

$$= \frac{1}{2}k_a\gamma' H^2 + \frac{1}{2}\gamma_w H^2$$

Here, γ' = Submerged unit weight

$= \gamma_{sat} - \gamma_w = 20 - 9.81 = 10.19 \text{ kN/m}^3$

$\therefore$ $$P_2 = \frac{1}{2} \times 0.271 \times 10.19 \times 5^2 + \frac{1}{2} \times 9.81 \times 5^2$$

$$= 34.52 + 122.63 = 157.15 \text{ kN/m}$$

$$\%\Delta P = \frac{P_2 - P_1}{P_1} \times 100 = \frac{157.15 - 57.58}{57.58} \times 100 = 172.92\% \text{ (increased)}$$

3. Backfill Subjected to Uniform Surcharge

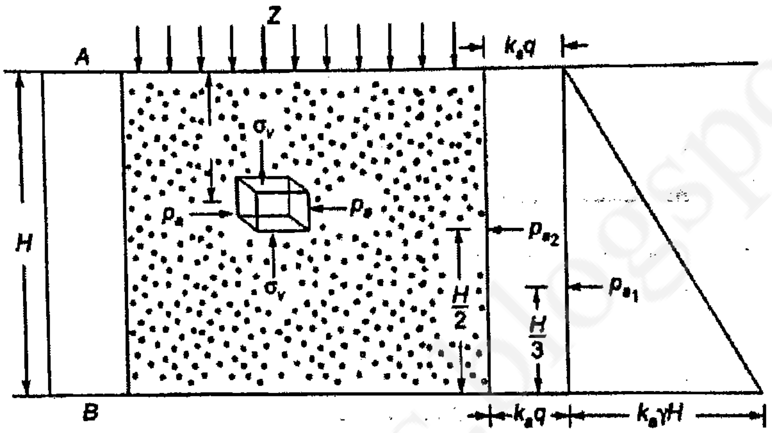


Fig. 11.17

The vertical stress at any depth z can be split into two viz. one due to soil grain and other due to uniform surcharge

$$\sigma_v = \gamma z + q$$

Let p_{a_1} = Active earth pressure due to soil grains

p_{a_2} = Active earth pressure due to uniform surcharge

Active pressure at any depth z from top,

$$p_a = k_a \sigma_v = k_a(\gamma z + q)$$

$$= k_a \gamma z + k_a q = p_{a_1} + p_{a_2}$$

- At point A ($z = 0$) $\quad p_{a_1} = 0$

  $p_{a_2} = k_a q$

  and $\quad p_a = 0 + k_a q = k_a q$

- At point B ($z = H$) $\quad p_{a_1} = k_a \gamma H$

  $p_{a_2} = k_a q$

  and $\quad p_a = k_a \gamma H + k_a q$

- Active thrusts, $\quad P_{a_1} = \frac{1}{2} k_a \gamma H^2$

$$\bar{h}_1 = \frac{H}{3} \text{ from base}$$

$$P_{a_2} = k_a q H$$

$$\bar{h}_2 = \frac{H}{2} \text{ form base}$$

Total active thrust, $$P_a = P_{a_2} + P_{a_2}$$

$$= \frac{1}{2} k_a \gamma H^2 + k_a q H$$

- The point of application of total thrust,

$$\bar{h} = \frac{P_{a_1}\bar{h}_1 + P_{a_2}\bar{h}_2}{P_{a_1} + P_{a_2}} = \frac{\Sigma P_{a_i}\bar{h}_i}{\Sigma P_{a_i}}$$

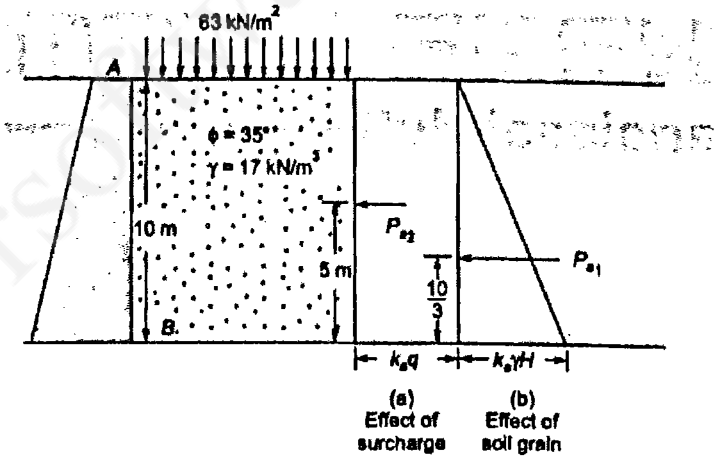
Example 11.5 A retaining wall 10 m high, has a smooth vertical back. The backfill has a horizontal surface in level with the top of the wall. There is a uniformly distributed surcharge of 63 kN/m^2 intensity over the backfill. The unit weight of backfill soil is 17 kN/m^3 with angle of shearing resistance, ϕ of 35° and cohesion is zero. Determine the magnitude and point of application of active pressure per meter length of wall.

Solution:

Coefficient of active earth pressure,

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 35°}{1+\sin 35°} = 0.271$$

Active pressure diagram:



I. **Effective of soil grains**

$$p_{a_1} = k_a \gamma z$$

At point A ($z = 0$), $p_{a_1} = 0$

At point B ($z = 10$ m), $p_{a_1} = 0.271 \times 17 \times 10 = 46.07$ kN/m^2

Total thrust contribution,

$$P_{a_1} = \frac{1}{2}\times 46.07\times 10\times 1 = 230.35 \text{ kN/m}$$

Point of application of P_{a_1} is at $\frac{H}{3} = \frac{10}{3}$ m from base.

II. Effect of uniform surcharge:

$$p_{a_2} = k_a q = 0.271\times 63 = 17.07 \text{ kN/m}^2 \text{ (constant)}$$

Total thrust contribution,

$$P_{a_2} = 17.07\times 10\times 1 = 170.70 \text{ kN/m}$$

Point of application of P_{a_2} is at $\frac{H}{2} = 5$ m from base

Total thrust on wall, $P_a = P_{a_1} + P_{a_2} = 230.35 + 170.70 = 401.05$ kN/m

Point of application of total thrust,

$$\bar{H} = \frac{P_{a_1}\bar{H}_1 + P_{a_2}\bar{H}_2}{P_{a_1} + P_{a_2}} = \frac{230.35\times\frac{10}{3} + 170.70\times 5}{401.05} = 4.04 \text{ m from base}$$

4. Partially Submerged Backfill:

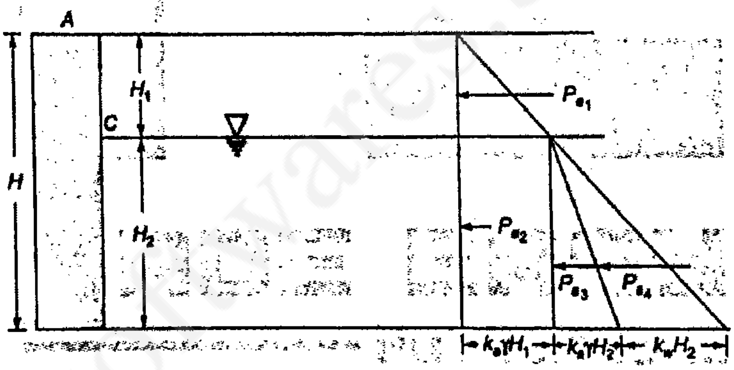


Fig. 11.18

Now this situation can be analyses by assuming separate soil layers of depth H_1 and H_2

For fist layer,

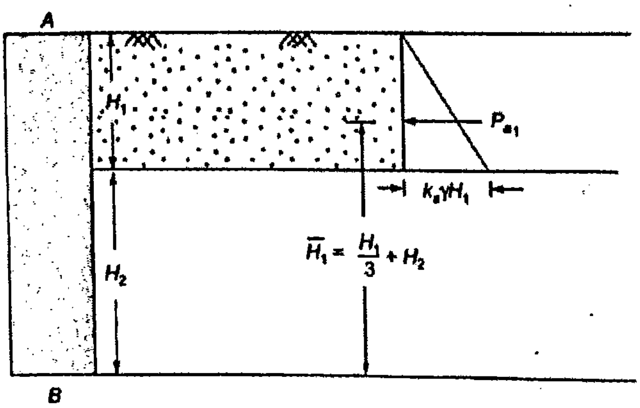


Fig. 11.19

At point C $(z = H_2)$ $\quad p_{a_4} = \gamma_w H_2$

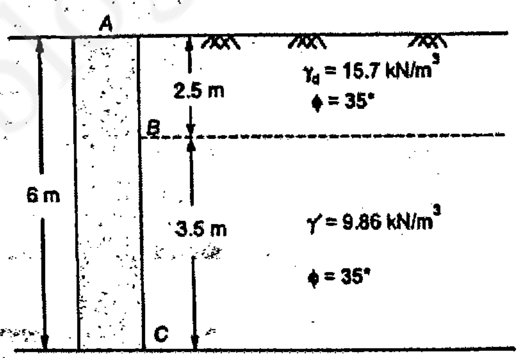
Total thrust contribution, $\quad P_{a_4} = \frac{1}{2} \times \gamma_w H_2 \times H_2 \times 1 = \frac{1}{2}\gamma_w H_2^2$

Line of action of P_{a_4}, $\quad \bar{H}_4 = \frac{H_2}{3}$

- Net thrust on the wall, $\quad P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4}$
- Line of action of total thrust, $\bar{H} = \frac{\Sigma P_{a_i} \bar{H}_i}{\Sigma P_{ai}}$

Example 11.6 **A retaining wall 6 m high supports earth with its face vertical. The earth is cohesionless with particle specific gravity 2.69, angle of internal friction 35° and porosity 40.5%. The surface is horizontal and level with the top of the wall. Determine the earth thrust and its line of action on the wall if the earth is water logged to level 2.5 m below the top surface. Neglect wall friction. Draw the pressure diagrams.**

Solution:



Given, $\quad H = 6$ m, $\phi = 35°$

Porosity, $\quad n = 40.5\%$, $G = 2.69$

We know that, $\quad n = \frac{e}{1+e}$

$$\therefore \quad e = \frac{n}{1-n} = \frac{0.405}{1-0.405} = 0.681$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.69 \times 9.81}{1+0.681} = 15.7 \text{ kN/m}^3$$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.69+0.681}{1+0.681}\right) \times 9.81 = 19.67 \text{ kN/m}^3$$

$$\therefore \quad \gamma' = \gamma_{sat} - \gamma_w = 19.67 - 9.81 = 9.86 \text{ kN/m}^3$$

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 35°}{1+\sin 35°} = 0.271$$

(i) Top soil layer AB:

$$p_{a_1} = k_a \gamma_d z$$

At point A $(z = 0)$, $\quad p_{a_1} = 0$

At point B $(z = 2.5$ m$)$, $p_{a_1} = 0.271 \times 15.7 \times 2.5 = 10.64$ kN/m²

Active thrust contribution

$$P_{a_1} = \frac{1}{2} p_{a_1} \times 2.5 \times 1 = 13.3 \text{ kN/m}$$

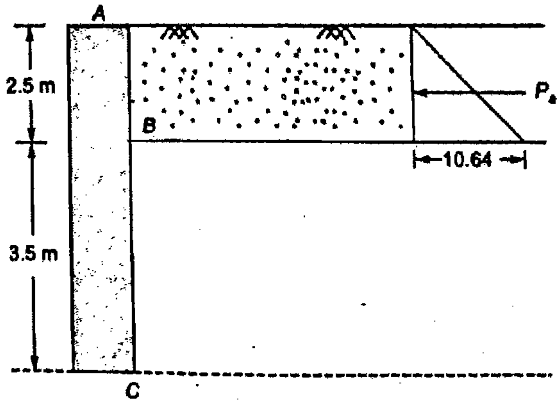


Fig. 11.21

Point of application of P_{a_1},

$$\bar{H}_1 = \frac{2.5}{3} + 3.5 \text{ m from base } C$$

(ii) Lower soil layer BC:

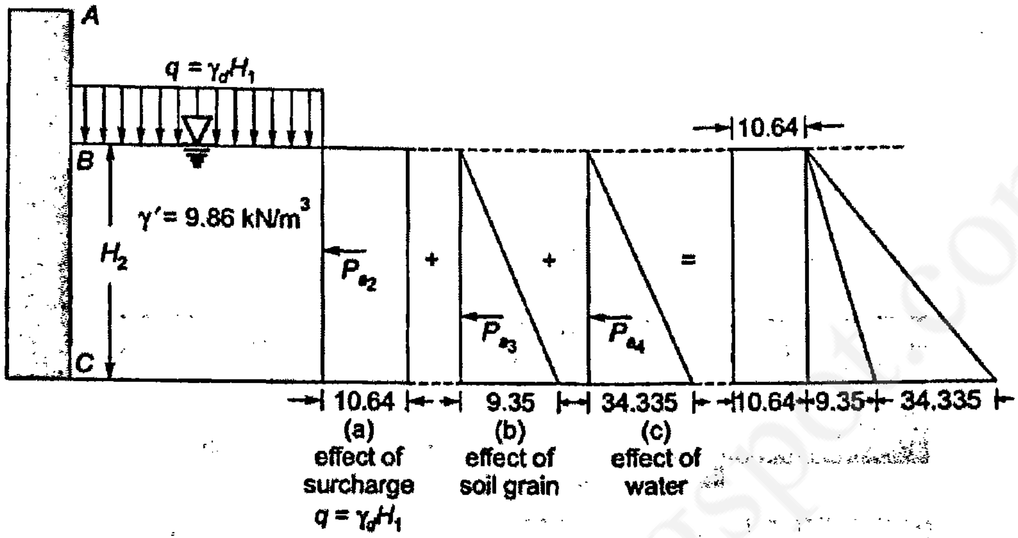


Fig. 11.22

(a) Effect of uniform surcharge:

$$q = \gamma_d H_1 = 15.7 \times 2.5 = 39.25$$

$$p_{a_2} = 0.271 \times 39.25 = 10.64 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{a_2} = p_{a_2} \times 3.5 = 10.64 \times 3.5 = 37.24$ kN/m

Line of action of P_{a_2}, $\bar{H}_2 = \frac{3.5}{2}$ from base C

(b) Effect of soil grain:

$$p_{a_3} = k_a \gamma z$$

At point B ($z = 0$),

$$p_{a_3} = 0.271 \times 9.86 \times 0 = 0$$

At point C ($z = 3.5$ m),

$$p_{a_3} = 0.271 \times 9.86 \times 3.5 = 9.35$$

Thrust contribution, $P_{a_3} = \frac{1}{2} \times 9.35 \times 3.5 \times 1 = 16.36$ kN/m²

Line of action of P_{a_3}, $\bar{H}_3 = \frac{3.5}{3}$ From base C

(c) Effect of water:

$$p_{a_4} = \gamma_w z$$

At point B ($z = 0$) $\quad p_{a_4} = 0$

At point C ($z = 3.5$), $\quad p_{a_4} = 9.81 \times 3.5 = 34.335$ kN/m²

Thrust contribution, $P_{a_4} = \frac{1}{2} \times 34.335 \times 3.5 \times 1 = 60.08$ kN/m

Line of action of P_{a_4}, $\bar{H}_4 = \frac{3.5}{3}$ from base C

Total thrust on wall, $P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4}$

$= 13.3 + 37.24 + 16.36 + 60.08$

$= 126.98$ kN/m

Line of action of total thrust,

$$\bar{H} = \frac{P_{a_1}\bar{H}_1 + P_{a_2}\bar{H}_2 + P_{a_3}\bar{H}_3 + P_{a_4}\bar{H}_4}{P_a}$$

$$= \frac{13.3 \times \left(\frac{2.5}{3} + 3.5\right) + 37.24 \times \frac{3.5}{2} + 16.36 \times \frac{3.5}{3} + 60.08 \times \frac{3.5}{3}}{126.98}$$

$= 1.67$ m from base

Pressure diagram:

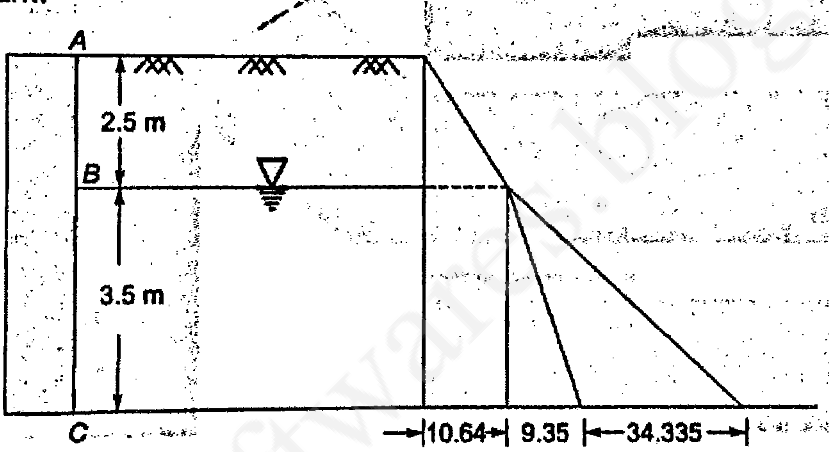
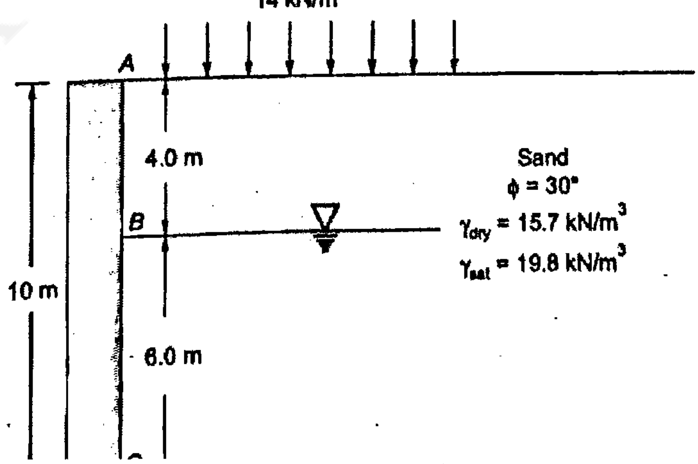


Fig. 11.23

Example 11.7 For an earth retaining wall shown in figure below, sketch the earth pressure m under active state and find total thrust per unit length of wall and its location.



$$\gamma' = \gamma_{sat} - \gamma_w = 19.8 - 9.81 = 9.99 \text{ kN/m}^3$$

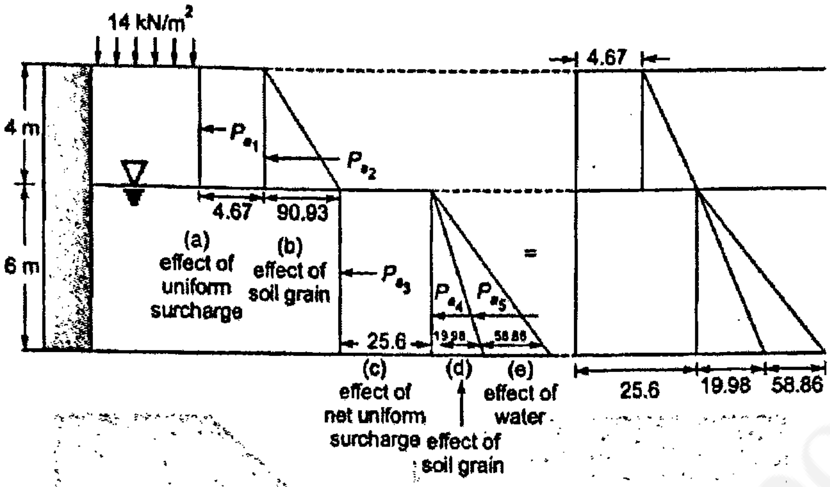


Fig. Pressure diagram

Top soil layer AB:

(i) Effect of uniform surcharge:

$$q = 14 \text{ kN/m}^2$$

$$p_{a_1} = k_a q = \frac{1}{3} \times 14 = 4.67 \text{ kN/m}^2 \text{ (Constant)}$$

Thrust contribution, $P_{a_1} = 4.67 \times 4 = 18.68$ kN/m

Line of action of P_{a_1}, $\bar{H}_1 = 2 + 6 = 8$ m from base

(ii) Effect of soil grains:

$$p_{a_2} = k_a \gamma_d z$$

At point A ($z = 0$), $p_{a_2} = \frac{1}{3} \times 15.7 \times 0 = 0$

At point B ($z = 4$ m) $p_{a_2} = \frac{1}{3} \times 15.7 \times 4 = 20.93$ kN/m²

Thrust contribution, $P_{a_2} = \frac{1}{2} \times 20.93 \times 4 = 41.86$ kN/m

Line of action of P_{a_2}, $\bar{H}_2 = \frac{4}{3} + 6 = 7.33$ m from base

Lower soil layer BC:

(i) Effect of uniform surcharge

$$q' = 14 + \gamma_d H_1 = 14 + 15.7 \times 4 = 76.8 \text{ kN/m}^2$$

$$p_{a_3} = k_a q' = \frac{1}{3} \times 76.8 = 25.6 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{a_3} = 25.6 \times 6 = 153.6$ kN/m

Line of action of P_{a_3}, $\bar{H}_3 = 3$ m from base

(ii) **Effect of soil grains**

$$p_{a_4} = k_a \gamma' z$$

At point B ($z = 0$), $p_{a_4} = 0$

At point C ($z = 6$ m), $p_{a_4} = \frac{1}{3} \times 9.99 \times 6 = 19.98 \text{ kN/m}^2$

Thrust contribution, $P_{a_4} = \frac{1}{2} \times 19.98 \times 6 = 59.94 \text{ kN/m}$

Line of action P_{a_4}, $\bar{H}_4 = \frac{6}{3} = 2$ m from base

(iii) **Effect of water**

$$p_{a_5} = \gamma_w z$$

At point B ($z = 0$), $p_{a_5} = 0$

At point C ($z = 6$ m) $p_{a_5} = 9.81 \times 6 = 58.86 \text{ kN/m}^2$

Thrust contribution, $P_{a_5} = \frac{1}{2} \times 58.86 \times 6 = 176.58 \text{ kN/m}$

Line of action P_{a_5}, $\bar{H}_5 = \frac{6}{3} = 2$ m from base

The total thrust,

$$P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4} + P_{a_5}$$
$$= 18.68 + 41.86 + 153.6 + 59.94 + 176.58$$
$$= 450.66 \text{ kN/m}$$

Line of action of total thrust,

$$\bar{H} = \frac{\sum P_{a_i} \bar{H}_i}{\sum P_{a_i}}$$
$$= \frac{18.68 \times 8 + 41.86 \times 7.33 + 153.6 \times 3 + 59.94 \times 2 + 176.58 \times 2}{450.66}$$
$$= 3.08 \text{ m from base}$$

5. Stratified backfill with different friction angles

Case-I: $\phi_1 > \phi_2$

(i) Active pressure and thrust

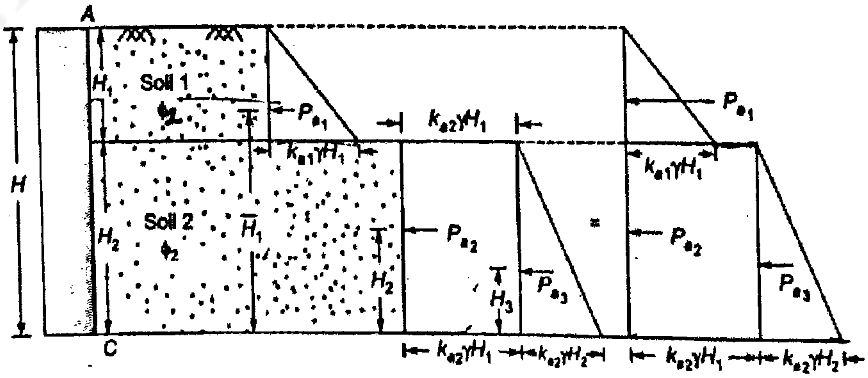


Fig. 11.24

Case-II: $\phi_1 < \phi_2$

- In this case, $k_{a_1} > k_{a_2}$ and $k_{p_1} < k_{p_2}$
- Active pressure diagrams

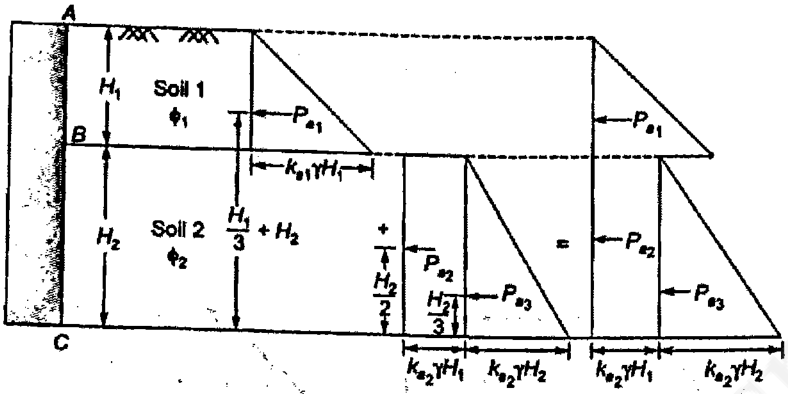


Fig. 11.26

- Passive pressure diagrams

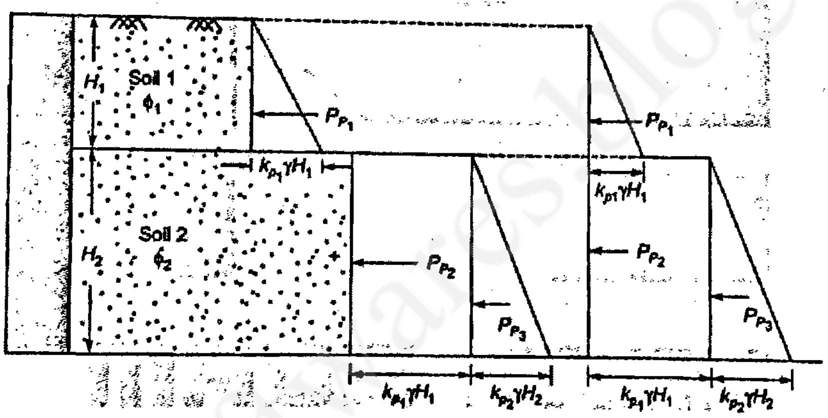
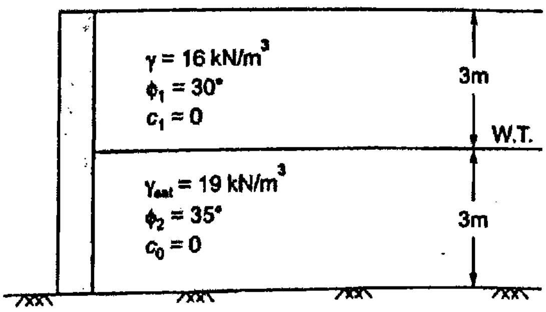


Fig. 11.27

Total active and passive thrust can be computed in similar manner as we calculated in previous case.

Example 11.8 For the retaining wall shown in the figure below, assume that the wall can yield sufficiently to develop active state. Use Rankine's active pressure theory and determine (a) active force per metre of the wall and (b) the location ofthe resultant line of action.

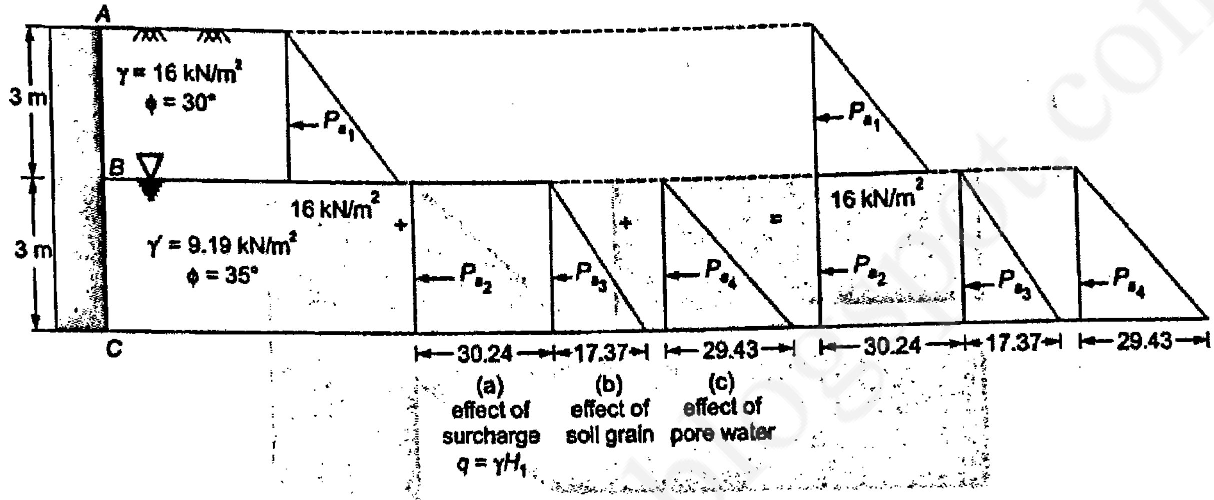


Solution:

$$k_{a_1} = \frac{1-\sin\phi_1}{1+\sin\phi_1} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

$$k_{a_2} = \frac{1-\sin\phi_2}{1+\sin\phi_2} = \frac{1-\sin 35°}{1+\sin 35°} = 0.271$$

$$\gamma' = \gamma_{sat} - \gamma_w = 19 - 9.81 = 9.19 \text{ kN/m}^3$$



Top soil layer AB:

$$p_{a_1} = k_{a_1}\sigma_v = k_{a_1}\gamma z$$

At point A ($z = 0$), $p_{a_1} = 0$

At point B ($z = 3$ m), $p_{a_1} = \frac{1}{3} \times 16 \times 3 = 16 \text{ kN/m}^2$

Thrust contribution, $P_{a_1} = \frac{1}{2} \times 16 \times 3 \times 1 = 24 \text{ kN/m}$

Line of action of P_{a_1}, $\bar{H}_1 = \frac{3}{3} + 3 = 4$ m from base

Lower soil layer BC:

(i) Effect of uniform surcharge due to top layer:

$$q = \gamma H_1 = 16 \times 3 = 48 \text{ kN/m}^2$$

$$p_{a_2} = k_{a_2} q = 0.271 \times 48 = 13.008 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{a_2} = 13.008 \times 3 = 39.024 \text{ kN/m}$

Line of action of P_{a_2}, $\bar{H}_2 = \frac{3}{2} = 1.5$ m from base

(ii) Effect of soil grains

$$p_{a_3} = k_{a_2}\sigma_v = k_{a_2}\gamma' z$$

At point B ($z = 0$), $p_{a_3} = 0$

At point C ($z = 3$ m), $p_{a_3} = 0.271 \times 9.19 \times 3 = 7.47$ kN/m$_2$

Thrust contribution, $P_{a_3} = \frac{1}{2} \times 7.47 \times 3 = 11.20$ kN/m

Line of action of P_{a_3}, $\bar{H}_3 = \frac{3}{3} = 1$ m from base

(iii) Effect of pore water

$$p_{a_4} = \gamma_w z$$

At point B ($z = 0$), $p_{a_4} = 0$

At point C ($z = 3$ m), $p_{a_4} = 9.81 \times 3 = 29.43$ kN/m^2

Thrust contribution, $P_{a_4} = \frac{1}{2} \times 29.43 \times 3 = 44.14$ kN/m

Line of action P_{a_4}, $\bar{H}_4 = \frac{3}{3} = 1$ m from base

Total active thrust per unit length of wall,

$$P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4}$$
$$= 24 + 39.024 + 11.20 + 44.14$$
$$= 118.37 \text{ kN/m}$$

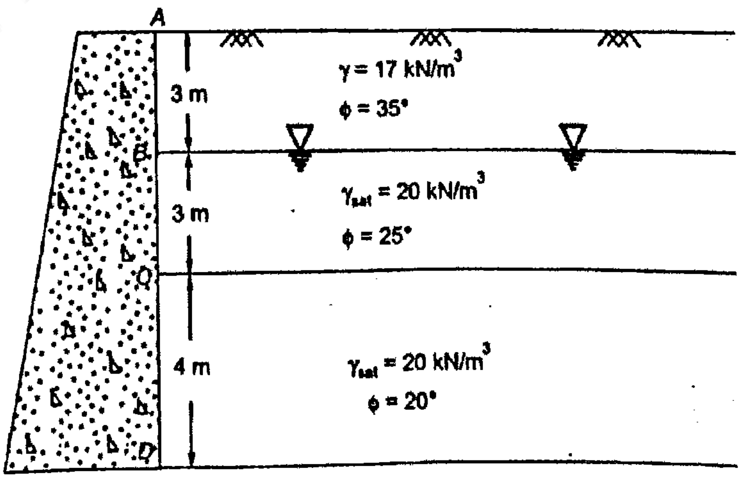
Line of action of total thrust,

$$\bar{H} = \frac{\sum P_{a_i} \bar{H}_i}{\sum P_{a_i}}$$

$$= \frac{24 \times 4 + 39.024 \times 1.5 + 11.20 \times 1 + 44.14 \times 1}{118.37}$$

$$= 1.773 \text{ m from base}$$

Example 11.9 For the retaining wall shown in figure below plot distribution of passive earth pressure and determine magnitude of total thrust and point of application of total thrust.



Solution:

Layer AB:

$$k_{p_1} = \frac{1+\sin 35°}{1-\sin 35°} = 3.69$$

$$p_{p_1} = k_{p_1}\sigma_v = k_{p_1}\gamma z$$

At point A ($z = 0$), $p_{p_1} = 0$

At point B ($z = 3$ m), $p_{p_1} = 3.69 \times 17 \times 3 = 188.19$ kN/m²

Thrust contribution, $P_{p_1} = \frac{1}{2} \times 188.19 \times 3 = 282.28$ kN/m

Line of action of P_{p_1}, $\bar{H}_1 = \frac{3}{3} + 7 = 8$ m (from base)

Layer BC:

$$k_{p_2} = \frac{1+\sin 25°}{1-\sin 25°} = 2.46$$

(i) Effect of uniform surcharge due to layer AB

$$q = \gamma_1 H_1 = 17 \times 3 = 51 \text{ kN/m}^2$$

$$p_{p_2} = k_{p_2} q = 2.46 \times 51 = 125.46 \text{ kN/m}^2$$

Thrust contribution, $P_{p_2} = 125.46 \times 3 = 376.38$ kN/m

Line of action of P_{p_2}, $\bar{H}_2 = \frac{3}{2} + 4 = 5.5$ m from base

(ii) Effect of soil grains

$$p_{p_3} = k_{p_2}\sigma_v = k_{p_2}\gamma' z$$

At point B ($z = 0$), $p_{p_3} = 0$

At point C ($z = 3$ m), $p_{p_3} = 2.46 \times (20 - 9.81) \times 3 = 75.20$ kN/m²

Thrust contribution, $P_{p_3} = \frac{1}{2} \times 75.20 \times 3 = 112.8$ kN/m

Line of action of P_{p_3}, $\bar{H}_3 = \frac{3}{3} + 4 = 5$ m from base

(ii) Effect of pore water

$$p_{p_4} = \gamma_w z$$

At point B ($z = 0$), $p_{p_4} = 0$

At point C ($z = 3$ m), $p_{p_4} = 9.81 \times 3 = 29.43$

Thrust contribution, $P_{p_4} = \frac{1}{2} \times 29.43 \times 3 = 44.14$ kN/m

Line of action of P_{p_4}, $\bar{H}_4 = \frac{3}{3} + 4 = 5$ m from base

Layer CD:

$$k_{p_3} = \frac{1+\sin 20^\circ}{1-\sin 20^\circ} = 2.04$$

(i) Effect of uniform surcharge due to layer AB and BC

$$q = \gamma_1 H_1 + \gamma'_2 H_2$$
$$= 17 \times 3 + (20 - 9.81) \times 3$$
$$= 81.57 \text{ kN/m}^2$$

$$p_{p_5} = k_{p_3} q = 2.04 \times 81.57 = 166.40 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{p_5} = 166.40 \times 4 = 665.6$ kN/m

Line of action of P_{p_5}, $\bar{H}_5 = \frac{4}{2} = 2$ m from base

(ii) Effect of soil grains

$$p_{p_6} = k_{p_3}\sigma_v = k_{p_3}\gamma' z$$

At point C ($z = 0$), $p_{p_6} = 0$

At point D ($z = 4$ m), $p_{p_6} = 2.04 \times (20 - 9.81) \times 4 = 83.15$ kN/m²

Thrust contribution, $P_{p_6} = \frac{1}{2} \times 83.15 \times 4 = 166.3$ kN/m

Line of action of P_{p_6}, $\bar{H}_6 = \frac{4}{3} = 1.33$ m from base

(iii) Effect of pore water

$$p_{p_7} = \gamma_w z$$

At point C ($z = 0$), $p_{p_7} = 0$

At point D ($z = 4$ m), $p_{p_7} = 9.81 \times 4$
$= 39.24$ kN/m²

Thrust contribution, $P_{p_7} = \frac{1}{2} \times 39.24 \times 4$
$= 78.48$ kN/m

Line of actio of P_{p_7}, $\bar{H}_7 = \frac{4}{3}$
$= 1.33$ m from base

Total passive thrust on wall

$$P_p = \sum P_{p_i}$$
$$= 282.28 + 376.38 + 112.8 + 44.14 + 665.6 + 166.3 + 78.48$$
$$= 1725.98 \text{ kN/m}$$

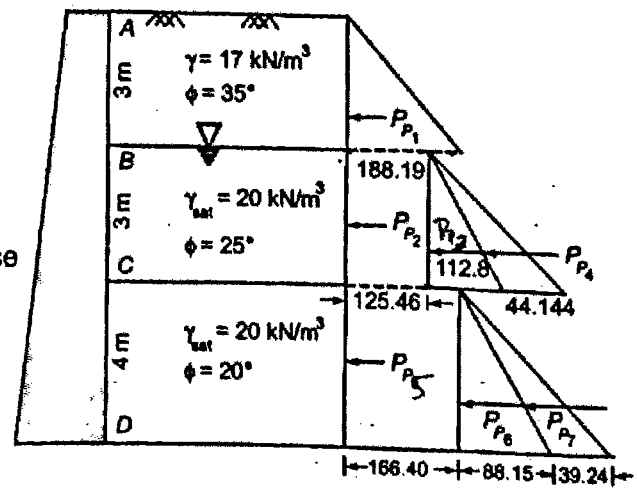


Fig.11.28

Line of action of total thrust, $\bar{H} = \dfrac{\sum P_{p_i}\bar{H}_i}{\sum P_{p_i}}$

$$= \frac{282.28\times8+376.38\times5.5+112.8\times5+44.14\times5+665.6\times2+166.3\times1.33+78.48\times1.33}{1725.98}$$

= 3.92 m from base

6. Inclined back with horizontal backfill

In this case the total lateral pressure on an imaginary vertical surface passing through the heel of the wall is found and is combined vectorially with the weight of the soil wedge between the imaginary face and the back of the wall, to give the resultant thrust on the wall.

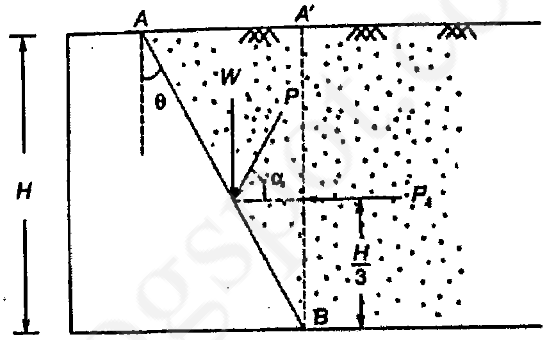


Fig. 11.29

- Let P_1 be the active or passive earth pressure on imaginary vertical plane (A' B) passing through heel of the wall.

$$P_1 = \frac{1}{2}k\gamma H^2$$

where k = coefficient of earth pressure (k_a or k_p)

- Resultant thrust on wall

$$P = \sqrt{W^2 + P_1^2}$$

- Angle of resultant with horizontal,

$$\tan\alpha = \frac{W}{P_1}$$

or

$$\alpha = \tan^{-1}\left(\frac{W}{P_1}\right)$$

where W = weight of the soil wedge between the imaginary face and the back of wall.

7. Vertical back with Sloping Surcharge

Rankine's theory for this case is based on the assumption that a 'conjugate' relationship exists between the vertical pressure and lateral pressure on vertical plane within the soil adjacent to a retaining wall.

The conjugate relationship direction of one stress is parallel to the plane on which the other acts.

The lateral pressure is act $\frac{H}{3}$ from base and is assumed to act parallel to the backfill surface.

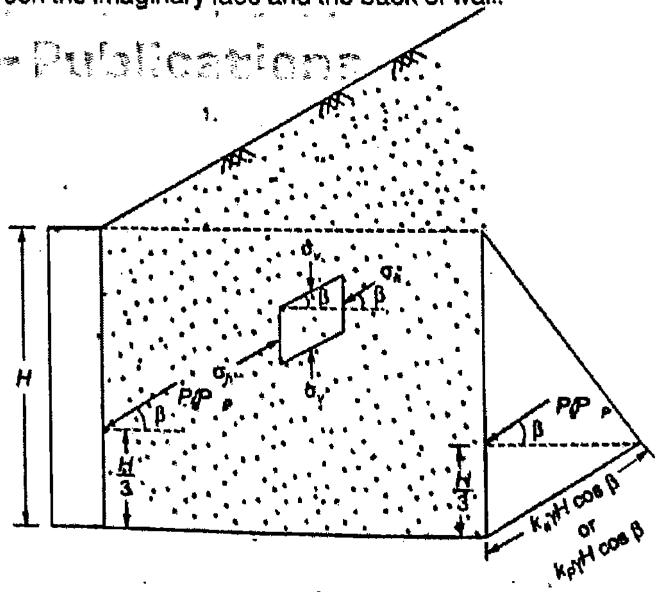


Fig. 11.30

- $p_a = k_a\gamma H\cos\beta$ and $p_p = k_p\gamma H\cos\beta$
- Thrust, $P_a = \frac{1}{2}k_a\gamma H^2\cos\beta$ and

$$P_p = \frac{1}{2} k_p \gamma H^2 \cos\beta$$

- Line of action of thrust $\frac{P_a}{P_p}$, $\bar{H} = \frac{H}{3}$ from base at an angle β with horizontal

- Where,

$$k_a = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$k_p = \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

8. Inclined back with sloping surcharge

Let P_1 be the active or passive earth pressure on imaginary vertical wall (CB) passing through heel of wall

$$P_1 = \frac{1}{2} k \gamma H^2 \cos\beta$$

where $k = k_a$ or k_p

$$k_a = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$k_p = \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

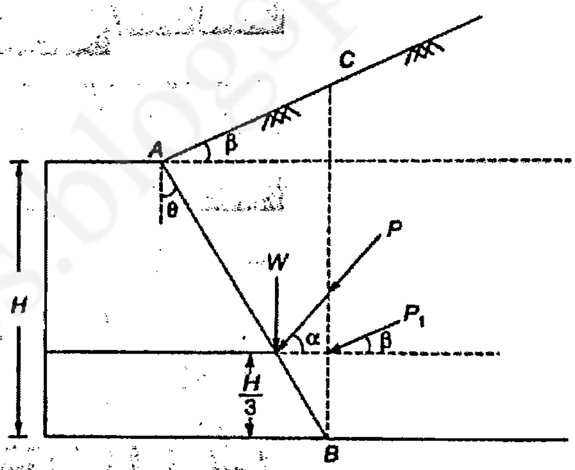


Fig. 11.31

- Horizontal component of thrust,

$$F_x = P_1 \cos\beta$$

and vertical component thrust,

$$f_y = W + P_1 \sin\beta$$

- Resultant thrust on back of wall,

$$P = \sqrt{F_x^2 + F_y^2} = \sqrt{(P_1\cos\beta)^2 + (W + P_1\cos\beta)^2}$$

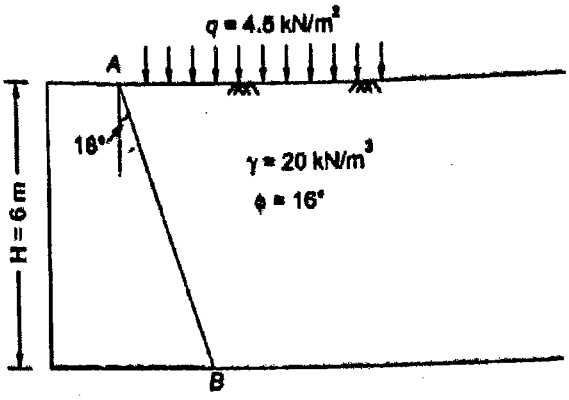
- Line of action of resultant thrust,

$$\tan\alpha = \frac{W + P_1\sin\beta}{P_1\cos\beta} = \frac{f_y}{f_x}$$

$$\alpha = \tan^{-1}\left(\frac{W + P_1\sin\beta}{P_1\cos\beta}\right) \text{ with the horizontal}$$

- Location of line of action, $\bar{H} = \frac{H}{3}$ from base at an angle α with the horizontal.

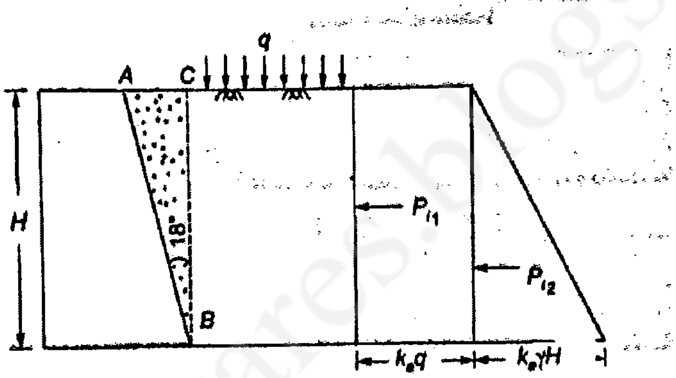
Example 11.10 **A soil mass is retained by a smooth incline wall of 6.0 m height. The soil has a bulk unit weight of 20 kN/m³ and ϕ = 16°. The top of the soil is level with the top of the wall and is horizontal. If soil surface carries a uniformly distributed load of 4.5 kN/m², determine the total active thrust on the wall per meter of the wall and its point of application.**



Solution:

Total active thrust on imaginary vertical wall (*BC*)

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 16^\circ}{1+\sin 16^\circ} = 0.568$$



(i). Effect of uniform surcharge

$$p_{h_1} = k_a q = 0.568 \times 4.5 = 2.56 \text{ kN/m}^2 \text{ (constant)}.$$

Thrust contribution, $P_{h_1} = 2.56 \times 6 = 15.36$ kN/m

Line of action of P_{h_1}, $\bar{H}_{h_1} = \frac{6}{2} = 3$ m from base

(ii). Effect of soil grains

$$p_{h_2} = k_a \sigma_v = k_a \gamma z$$

At point C ($z = 0$) $p_{h_2} = 0$

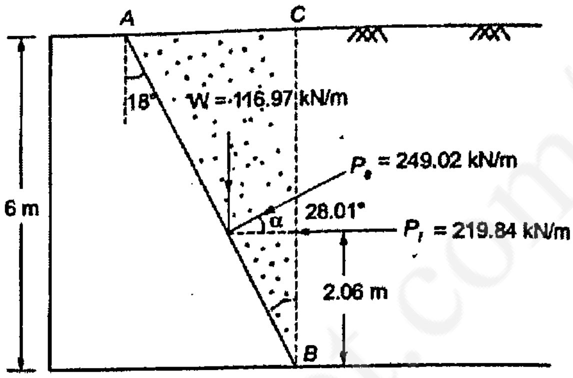
At point B ($z = 6$ m), $p_{h_2} = 0.568 \times 20 \times 6 = 68.16$ kN/m²

Thrust contribution, $P_{h_2} = \frac{1}{2} \times 68.16 \times 6 = 204.48$ kN/m²

Line of action of P_{h_2}, $\bar{H}_{h_2} = \frac{6}{3} = 2$ m from base

Total active thrust on imaginary vertical wall (CB),

$$P_l = P_{l_1} + P_{l_2}$$
$$= 15.36 + 204.48$$
$$= 219.84 \text{ kN/m}$$



Line of action of P_l,

$$\bar{H}_l = \frac{\sum P_l \bar{H}_l}{\sum P_l}$$
$$= \frac{15.36 \times 3 + 204.48 \times 2}{219.84}$$
$$= 2.07 \text{ m}$$

Weight of soil within wedge ABC,

$$W = \frac{1}{2} \times AC \times CB \times 1 \times \gamma = \frac{1}{2} \times 6 \tan 18° \times 6 \times 1 \times 20$$
$$= 116.97 \text{ kN/m}$$

Resultant thrust on wall

$$P_a = \sqrt{P_l^2 + W^2} = \sqrt{(219.84)^2 + (116.97)^2}$$
$$= 249.02 \text{ kN/m}^2$$

Angle of inclination of total thrust,

$$\alpha = \tan^{-1}\left(\frac{W}{P_l}\right) = \tan^{-1}\left(\frac{116.97}{219.84}\right) = 28.01°$$

11.8 Active and Passive Earth Pressure in Cohesive Soils

Cohesive soils are partially self-supporting. Pressure exerted by these soils on the retaining structure in active state is comparatively less than the cohesionless soil but the pressure exerted by these soils in passive state is comparatively more than the cohesionless soils.

Originally, the Rankine's theory considered the case of only cohesionless soil but this theory been extended by Bell to cover the case of C-ϕ backfill.

Consider a retaining wall of height H with a smooth vertical back, retaining a cohesive backfill.

The relationship between the major principal stress σ_1 and minor principal stress σ_3 at failure (Plastic equilibrium) can be expressed in the form

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} \quad \text{...(i)}$$

11.8.1 Active State

In active state, lateral stress σ_h reduces to its minimum value i.e., p_a while the vertical stress σ_v remains unchanged.

Since, $\sigma_v > \sigma_h$

Hence, $\sigma_1 = \sigma_v = \gamma z$

and $\sigma_3 = \sigma_h = p_a$

Substituting the values of σ_1 and σ_3 in eq. (i), we get

$$\sigma_v = \sigma_h\left(\frac{1+\sin\phi}{1-\sin\phi}\right)+2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\gamma z = p_a\left(\frac{1+\sin\phi}{1-\sin\phi}\right)+2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

On rearranging,

$$p_a = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)\gamma z-2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

$$p_a = k_a\gamma z-2c\sqrt{k_a} \qquad \left[\because k_a = \frac{1-\sin\phi}{1+\sin\phi}\right]$$

At point A ($z = 0$), $p_a = -2c\sqrt{k_a}$

At point B ($z = H$), $p_a = k_a\gamma H-2c\sqrt{k_a}$

Negative pressure in cohesive soils in active state at top indicates that these soils are in the state of tension and these tensile stresses gradually reduces to zero at some depth z_0 below the ground surface.

$$p_a = 0, \quad \text{when } k_a\gamma z_0-2c\sqrt{k_a} = 0$$

$$2c\sqrt{k_a} = k_a\gamma z_0$$

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}}$$

For pure clays, $\phi = 0$

Hence $k_a = 1$

$$\therefore \quad z_0 = \frac{2c}{\gamma}$$

It is also clear that the net active thrust is zero for a depth equal to $2z_0$. Thus, it implied that in a cohesive soil, a vertical cut can be made upto a depth of $2z_0$ without providing any lateral support. This depth of $2z_0$ is referred as critical depth of unsupported cut.

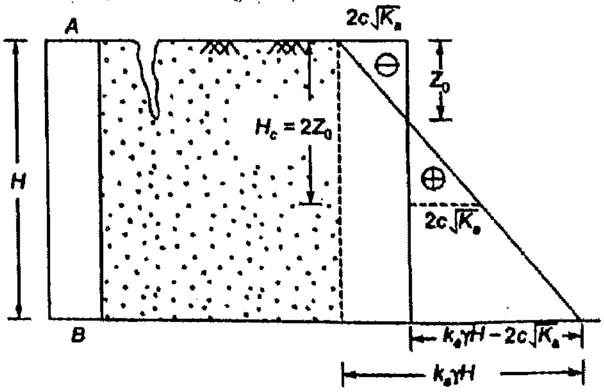


Fig. 11.32

$$H_c = 2z_0 = \frac{4c}{\gamma\sqrt{k_a}}$$

$$= \frac{4c}{\gamma}\tan\left(45^\circ+\frac{\phi}{2}\right)$$

For pure cohesive soils ($\phi = 0$)

$$H_c = \frac{4c}{\gamma}$$

Example 11.11 An unsupported excavation is to be made in a clay layer. If γ_t = 18 kN/m³, c = 30 kN/m² and ϕ = 10°.

(a) Calculate the depth of tension cracks;

(b) Calculate the maximum possible unsupported depth; and

(c) Draw the active pressure distribution diagram.

Solution:

(a) Depth of tension cracks is given by

$$z_0 = \frac{2c}{\gamma\sqrt{k_A}} \qquad \text{Where } k_A = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 10°}{1+\sin 10°} = \frac{0.826}{1.174} = 0.704$$

$$\gamma = \gamma_t = 18 \text{ kN/m}^3$$

$$\therefore \quad z_0 = \frac{2\times 30}{18\sqrt{0.704}} = 3.973 \text{ m}$$

(b) The safe depth of unsupported excavation in clay is given by

$$= \frac{2z_0}{F} \text{ where F is factor of safety}$$

∴ Max. depth of unsupported excavation = $2z_0$ (i.e., when F = 1)

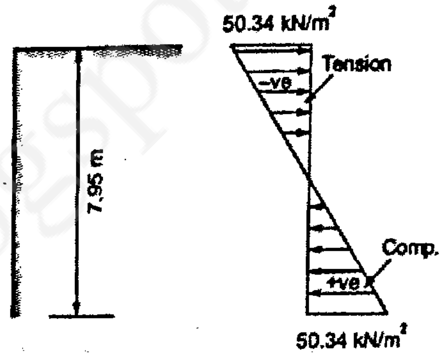
$$= 2 \times 3.973 = 7.95 \text{ m}$$

(c) Compute maximum active pressure (–ve at top)

$$= 2c \times \sqrt{k_A}$$

$$= 2\times 30\times\sqrt{0.704}$$

$$= 50.34 \text{ kN/m}^2$$



Active pressure diagram

The active pressure diagram along the unsupported maximum, depth is shown in figure.

11.8.2 Total active pressure per unit length of wall

(a) When tension cracks do not develop

- In this case, for top z_0 depth, soil will remain just to the contact of wall. Hence net thrust contribution of depth $2z_0$ will be zero.

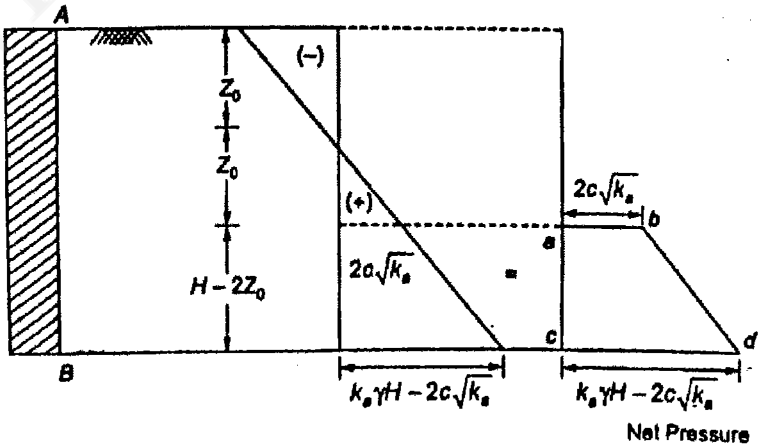


Fig. 11.33

Scanned by CamScanner

- $$P_a = \frac{1}{2}(ab+cd)\times(H-2z_0)\times 1$$

$$= \frac{1}{2}\left[2c\sqrt{k_a}+k_a\gamma H-2c\sqrt{k_a}\right]\times(H-2z_0)$$

$$= \frac{1}{2}k_a\gamma H\left[H-\frac{4c}{\gamma\sqrt{k_a}}\right] = \frac{1}{2}k_a\gamma H^2-2cH\sqrt{k_a}$$

- Line of action of P_a, $$\bar{H} = \left(\frac{cd+2ab}{cd+ab}\right)\times\frac{H-2z_0}{3}$$

$$= \left[\frac{k_a\gamma H-2c\sqrt{k_a}+4c\sqrt{k_a}}{k_a\gamma H-2c\sqrt{k_a}+2c\sqrt{k_a}}\right]\times\frac{H-2z_0}{3} \text{ from base}$$

- Above results can be found by using integration

$$P_a = \int_0^H p_a \times dz \times 1 = \int_0^H \left(k_a\gamma H-2c\sqrt{k_a}\right)dz$$

$$= k_a\gamma\left[\frac{z^2}{2}\right]_0^H - 2c\sqrt{k_a}\,[z]_0^H$$

$$= \frac{1}{2}k_a\gamma H^2-2cH\sqrt{k_a}$$

(b) When tension cracks develops:

- In this case, soil will not be in contact with the wall upto z_0. Hence wall will be free from any thrust due to negative pressure developed in soil within z_0. The resultant active thrust will be due to remaining $(H-z_0)$ depth of soil.

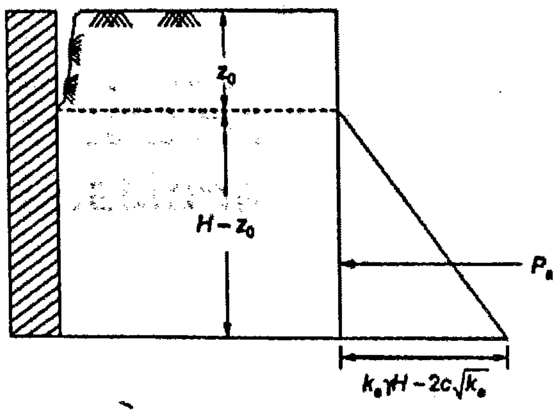


Fig. 11.34

$$P_a = \frac{1}{2}\left(k_a\gamma H-2c\sqrt{k_a}\right)\times(H-z_0)$$

$$= \frac{1}{2}\left(k_a\gamma H-2c\sqrt{k_a}\right)\times\left(H-\frac{2c}{\gamma\sqrt{k_a}}\right)$$

$$= \frac{1}{2}\left(k_a\gamma H^2-2cH\sqrt{k_a}-2c\sqrt{k_a}\,H+\frac{4c^2}{\gamma}\right)$$

$$= \frac{1}{2}k_a\gamma H^2-2c\sqrt{k_a}\,H+\frac{2c^2}{\gamma}$$

- Line of action of P_a,

$$\bar{H} = \frac{H-z_0}{3} \text{ from base}$$

- Above results can be found by using integration

$$P_a = \int_{z_0}^{H} p_a \times dz \times 1 = \int_{z_0}^{H} \left(k_a \gamma z - 2c\sqrt{k_a}\right) dz$$

$$= k_a \gamma \left[\frac{z^2}{2}\right]_{z_0}^{H} - 2c\sqrt{k_a}\,[z]_{z_0}^{H}$$

$$= \frac{1}{2}k_a \gamma H^2 - \frac{1}{2}\gamma z_0^2 - 2c\sqrt{k_a}H + 2c\sqrt{k_a}z_0$$

$$= \frac{1}{2}k_a \gamma H^2 - \frac{1}{2}\gamma \frac{4c^2}{\gamma^2 k_a} - 2c\sqrt{k_a}H + 2c\sqrt{k_a}\frac{2c}{\gamma\sqrt{k_e}}$$

$$= \frac{1}{2}k_a \gamma H^2 - 2c\sqrt{k_a}H - \frac{2c^2}{\gamma} + \frac{4c^2}{\gamma}$$

$$= \frac{1}{2}k_a \gamma H^2 - 2c\sqrt{k_a}H + \frac{2c^2}{\gamma}$$

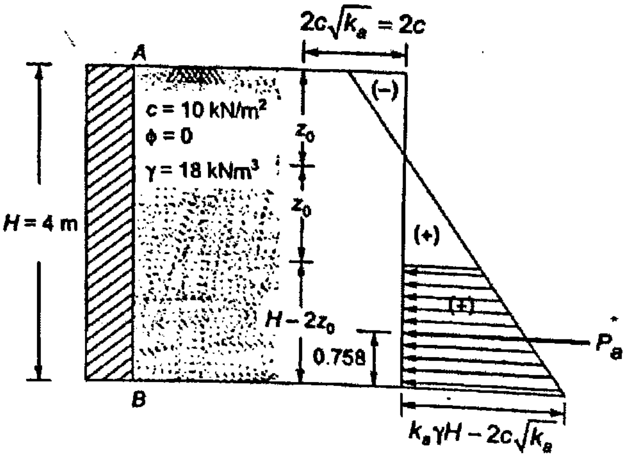
NOTE: In general, in calculating the total active thrust on the wall, the tension zone is usually ignored and only the area of the pressure distribution between the depth z_0 and H is considered.

Example 11.12 A smooth vertical wall 4 m retains cohesive soil backfill with $c = 10$ kN/m², $\phi = 0$ and $\gamma = 18$ kN/m². Determine

(i) The depth at which active earth pressure is zero.
(ii) Depth of tension crack
(iii) Depth at which total active thrust is zero
(iv) Plot of active pressure distribution
(v) Total active thrust per unit length of wall when tension cracks not developed.

Solution:

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-0}{1+0} = 1$$



(i) The active pressure at any depth z from surface is given by

$$p_a = k_a \gamma z - 2c\sqrt{k_a}$$

For p_a to be zero

$$k_a \gamma z - 2c\sqrt{k_a} = 0$$

$$z = \frac{2c\sqrt{k_a}}{\gamma k_a} = \frac{2c}{\gamma\sqrt{k_a}}$$

$$= \frac{2\times10}{18\times1} = 1.11 \text{ m from surface}$$

(ii) Depth of tension cracks

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}} = \frac{2\times10}{18\times1} = 1.11 \text{ m from surface}$$

(iii) Net active thrust will be zero upto a depth of $2z_0$

$$H_c = 2z_0 = \frac{4c}{\gamma\sqrt{k_a}} = \frac{4\times10}{18\times1} = 2.22 \text{ m from surface}$$

(iv) Active pressure diagram

$$p_a = k_a\gamma z - 2c\sqrt{k_a}$$

At point A ($z = 0$),

$$p_a = -2c\sqrt{k_a} = -2\times10\times1 = -20 \text{ kN/m}^2$$

At point B ($z = 4$ m),

$$p_a = 1\times18\times4 - 2\times10\times1 = 52 \text{ kN/m}^2$$

Position of zero pressure,

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}} = \frac{2\times10}{18\times1} = 1.1 \text{ m}$$

Active pressure diagram is shown in figure.

(v) Pressure at $2z_0$ from surface

$$p_a = k_a\gamma\times2.22 - 2c\sqrt{k_a}$$
$$= 1\times18\times2.22 - 2\times10\times1$$
$$= 20 \text{ kN/m}^2$$

Total active thrust,

$$P_a = \frac{1}{2}(20+52)\times(H-2z_0)$$

$$= \frac{1}{2}(20+52)\times(4-2\times1.11) = 64.08 \text{ kN/m}$$

Line of action of total active thrust,

$$\bar{H} = \left[\frac{52+2\times20}{52+20}\right]\times\frac{(H-2z_0)}{3}$$

$$= \left[\frac{52+2\times20}{52+20}\right]\times\frac{(4-2\times1.11)}{3} = 0.758 \text{ m from base}$$

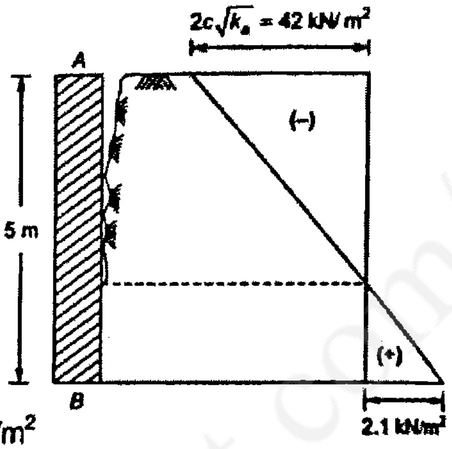
Example 11.13 A 5 m high smooth retaining wall with vertical face retains a cohesive backfill having $c = 30$ kN/m², $\gamma = 18$ kN/m² and $\phi = 20°$. Calculate the depth of tension crack and the total active thrust, assuming the tension cracks has fully developed. The backfill surface is horizontal.

Solution:

Given:

$$c = 30\ \text{kN/m}^2$$
$$\gamma = 18\ \text{kN/m}^3$$
$$\phi = 20°$$

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 20°}{1+\sin 20°} = 0.49$$



Active pressure at any depth z below soil surface is given by

$$p_a = k_a\sigma_v - 2c\sqrt{k_a} = k_a\gamma z - 2c\sqrt{k_a}$$

At point A ($z = 0$), $p_a = -2c\sqrt{k_a} = -2\times 30\times\sqrt{0.49} = -42\ \text{kN/m}^2$

At point B ($z = 5$ m), $p_a = 0.49\times 18\times 5 - 2\times 30\times\sqrt{0.49} = 2.1\ \text{kN/m}^2$

Since active pressure change sign, hence there must be a point of zero pressure

$$\therefore \quad k_a\gamma z_0 - 2c\sqrt{k_a} = 0$$

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}} = \frac{2\times 30}{18\times\sqrt{0.49}} = 4.76\ \text{m from top}$$

(i) Depth of tension crack

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}} = 4.76\ \text{m from top}$$

(ii) Total active thrust, when tension cracks has been fully developed

$$P_a = \frac{1}{2}\times 2.1\times(H - z_0)\times 1$$

$$= \frac{1}{2}\times 2.1\times(5-4.76)\times 1 = 0.252\ \text{kN/m}$$

(c) Backfill subjected to uniform surcharge

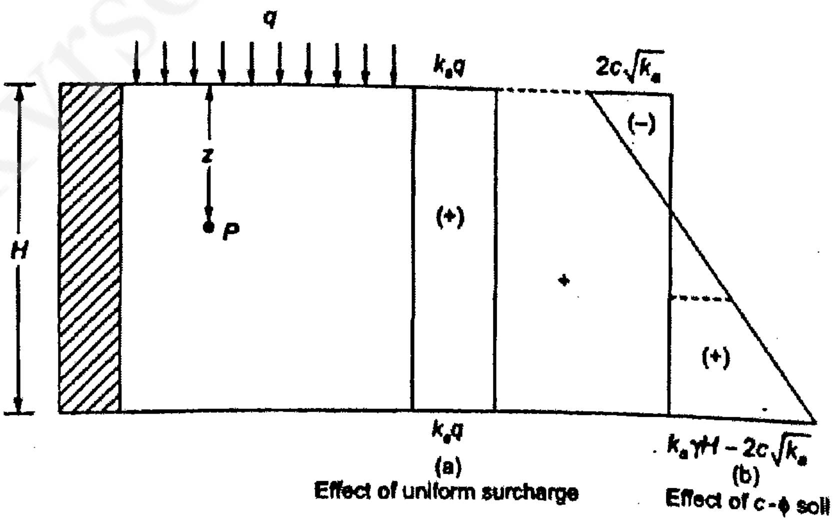


Fig. 11.35

We know active pressure at any depth 'z' below top is given by

$$p_a = k_a\sigma_v - 2c\sqrt{k_a}$$

where σ_v = vertical stress at a depth 'z'
$= (q + \gamma z)$

$$\therefore \quad p_a = k_a(q+\gamma z) - 2c\sqrt{k_a}$$

$$= k_a q + \left(k_a\gamma z - 2c\sqrt{k_a}\right)$$

$$p_a = p_{a_1} + p_{a_2}$$

where, p_{a_1} = Active pressure due to uniform surcharge

p_{a_2} = Active pressure due to c-ϕ soil

In case of uniform surcharge, tension crack may or may not be develop. It will depend upon the magnitude of q.

Example 11.14 A retaining wall with a smooth vertical back face has to retain a backfill of c-ϕ soil upto 5 m above the ground level. The surface of the backfill is horizontal and it has following properties.

γ = 18 kN/m³, cohesion = 15 kN/m², ϕ = 12°

(a) Plot the distribution of the active pressure on the wall.
(b) Determine the depth of tension cracks zone.
(c) Determine the magnitude and point of application of the active thrust.
(d) Determine the intensity of a fictitious uniform surcharge which is placed over the backfill which can prevent the formation of tension cracks.
(e) Compute the resultant active thrust after placing the surcharge.

Solution:

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 12^\circ}{1+\sin 12^\circ}$$

$$= 0.656$$

(a) Active pressure at any distance z below top,

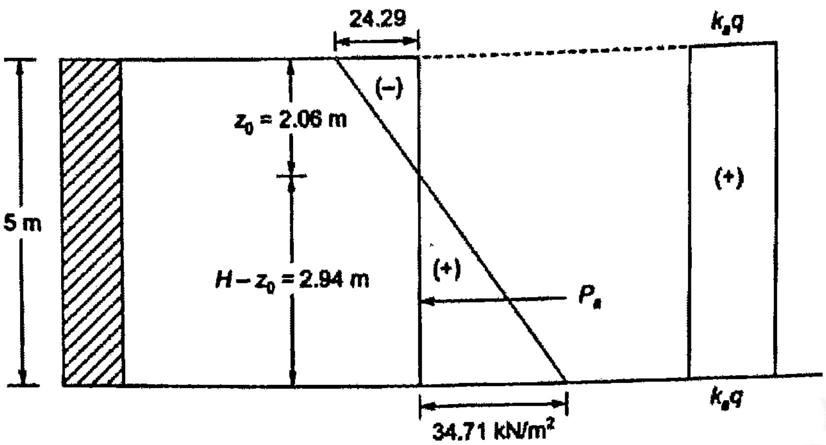
$$p_a = k_a\gamma z - 2c\sqrt{k_a}$$

At $z = 0$, $\quad p_a = 0 - 2\times 15\times\sqrt{0.656}$
$= -24.29$ kN/m²

At $z = 5$ m, $\quad p_a = 0.656\times 18\times 5 - 2\times 15\times\sqrt{0.656}$
$= 34.71$ kN/m²

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}} = \frac{2\times 15}{18\sqrt{0.656}}$$

$$= 2.06 \text{ m}$$



(b) Depth of tension cracks,

$$z_0 = \frac{2c}{\gamma\sqrt{k_a}} = 2.06 \text{ m}$$

(c) Total active thrust, ignoring tension zone,

$$P_a = \frac{1}{2} \times 34.71 \times 2.94 \times 1 = 51.024 \text{ kN/m}$$

Line of action of P_a, $\bar{H} = \frac{H - z_0}{3} = \frac{2.94}{3} = 0.98$ m

(d) Let 'q' be the fictitious surcharge, after application of surcharge pressure is given as

$$p_a = k_a \sigma_v - 2c\sqrt{k_a}$$

$$= k_a(q - \gamma z) - 2c\sqrt{k_a}$$

At $z = 0$, $p_a = k_a q - 2c\sqrt{k_a}$

For no tension at top,

$$p_a = k_a q - 2c\sqrt{k_a} = 0$$

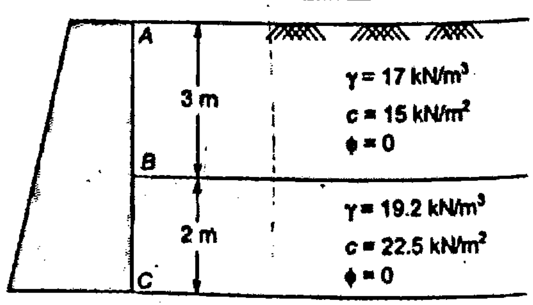
$$q = \frac{2c\sqrt{k_a}}{k_a} = \frac{2c}{\sqrt{k_a}} = \frac{2 \times 15}{\sqrt{0.656}} = 37.04 \text{ kN/m}^2$$

(e) $$P_a = \frac{1}{2} \times 34.71 \times 2.94 \times 1 + k_a q \times H$$

$$= 51.024 + 0.656 \times 37.04 \times 5$$

$$= 172.474 \text{ kN/m}$$

Example 11.15 A 5 m high vertical wall supports a saturated cohesive soil ($\phi = 0$) with horizontal backfill, the top 3 m backfill has unit weight of 17 kN/m³ and cohesion of 15 kN/m². The bulk unit weight and cohesion of the lower backfill of 2 m is 19.2 kN/m³ and 22.5 kN/m² respectively. If tension cracks developed then what would be the total active thrust on the wall. Also draw the pressure distribution diagram.



Solution:

Top soil layer *AB*:

$$k_{a_1} = \frac{1-\sin\phi_1}{1+\sin\phi_1} = \frac{1-0}{1+0} = 1$$

$$p_{a_1} = k_{a_1}\gamma z - 2c\sqrt{k_a}$$

At point A ($z = 0$), $p_{a_1} = 1\times17\times0-2\times15\sqrt{1} = -30$ kN/m²

At point B ($z = 3$ m), $p_{a_1} = 1\times17\times3-2\times15\sqrt{1} = 21$ kN/m²

Depth of tension cracks, $z_0 = \frac{2c}{\gamma\sqrt{k_{a_1}}} = \frac{2\times15}{17\times\sqrt{1}} = 1.76$ m

Thrust contribution, $P_{a_1} = \frac{1}{2}\times21\times(H_1-z_0)\times1 = \frac{1}{2}\times21\times(3-1.76)\times1$

$= 13.02$ kN/m

Line of action of P_{a_1}, $\bar{H}_1 = \left(\frac{3-1.76}{3}\right)+2 = 2.41$ m from base

Bottom soil lower BC:

$$k_{a_2} = \frac{1-\sin\phi_2}{1+\sin\phi_2} = \frac{1-\sin0^\circ}{1+\sin0^\circ} = 1$$

(i) Effect of uniform surcharge due to top soil layer AB

$$q = \gamma_1H_1 = 17\times3 = 51 \text{ kN/m}^2$$

$$p_{a_2} = k_{a_2}q = 1\times51 = 51 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{a_2} = 51\times2 = 102$ kN/m

Line of action of P_{a_2}, $\bar{H}_2 = \frac{2}{2} = 1$ m from base

(ii) Effect of soil grain of soil mass:

$$p_{a_3} = k_{a_2}\gamma z - 2c\sqrt{k_{a_2}}$$

At point B ($z = 0$), $p_{a_3} = 0-2c\sqrt{k_{a_2}} = -2\times22.5\sqrt{1} = -45$ kN/m²

At point C ($z = 2$ m), $p_{a_3} = 1\times19.2\times2-2\times22.5\times\sqrt{1} = -6.6$ kN/m²

Thrust contribution, $P_{a_3} = -\frac{(45+6.6)}{2}\times2 = -51.6$ kN/m

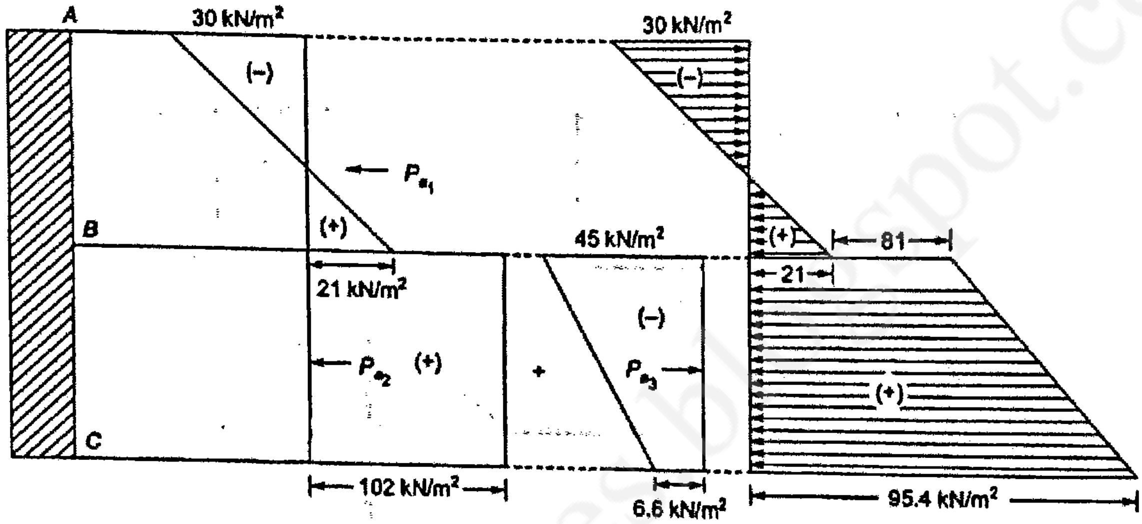
Line of action of P_{a_3}, $\bar{H}_3 = \left(\frac{6.6+2\times45}{6.6+45}\right)\times\frac{2}{3} = 1.24$ m from base

Total thrust per unit length of wall,

$$P_a = P_{a_1} + P_{a_2} + P_{a_3}$$
$$= 13.02 + 102 - 51.6$$
$$= 63.42 \text{ kN/m}$$

Line of action of total thrust,

$$\bar{H} = \frac{\sum P_{a_i}\bar{H}_i}{\sum P_{a_i}} = \frac{13.02 \times 2.41 + 102 \times 1 - 51.6 \times 1.24}{63.42} = 1.09 \text{ m from base}$$



Example 11.16 A retaining wall 4 m high with a smooth vertical back is pushed against a soil mass having $c = 20$ kN/m² and $\phi = 20°$ and $\gamma = 19.2$ kN/m³. Using Rankine's theory compute total pressure and the point of application of resultant thrust, if horizontal soil surface carries a uniform surcharge of 60 kN/m².

Solution:

$$k_p = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 20°}{1-\sin 20°} = 2.04$$

(i) Effect of uniform surcharge

$$p_{p_1} = k_p q = 2.04 \times 60 = 122.4 \text{ kN/m}^2 \text{ (constant)}$$

Passive thrust contribution,

$$P_{p_1} = 122.4 \times 4 = 489.6 \text{ kN/m}$$

Line of action of P_{p_1}, $\bar{H}_1 = \frac{4}{2} = 2$ m from base

(ii) Effect of c-ϕ soil mass

(a) Due to ϕ-component of soil

$$p_{p_2} = k_p \gamma z$$

At point A ($z = 0$), $p_{p_2} = 0$

At point B ($z = 4$m), $p_{p_2} = 2.04 \times 19.2 \times 4 = 156.67$ kN/m²

Thrust contribution, $P_{p_2} = \frac{1}{2} \times 156.67 \times 4 = 313.34$ kN/m

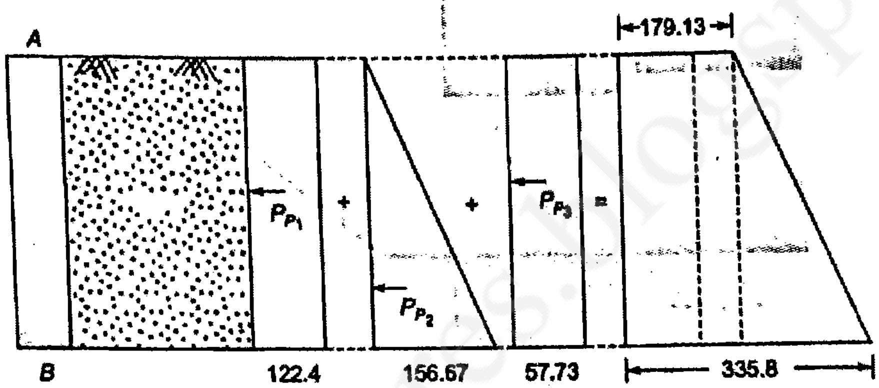
Line of action of P_{p_2}, $\bar{H}_2 = \frac{4}{3}$ From base

(b) Due to c-component of soil,

$$p_{p_3} = 2c\sqrt{k_p} = 2 \times 20 \times \sqrt{2.04} = 57.13 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{p_3} = 57.13 \times 4 = 228.52$ kN/m

Line of action of P_{p_3}, $\bar{H}_3 = \frac{4}{2} = 2$ m from base



Total passive thrust per unit length of wall,

$$P_p = P_{p_1} + P_{p_2} + P_{p_3}$$
$$= 489.60 + 313.34 + 228.52$$
$$= 1031.46 \text{ kN/m}$$

Line of action of P_p, $\bar{H} = \frac{\sum P_{p_i} \bar{H}_i}{\sum P_{p_i}} = \frac{489.60 \times 2 + 313.34 \times \left(\frac{4}{3}\right) + 228.52 \times 2}{1031.46}$

$= 1.78$ m from base

11.9. Coulomb's Wedge Theory

Coulomb's theory of earth pressure involves the consideration of a sliding wedge which tends to break away from the rest of the backfill upon wall movement. When the wall moves outward, the sliding wedge moves downwards and outwards; the sliding wedge moves upward and inwards when the wall is pushed toward the backfill.

Assumptions:

1. Soil is homogeneous, isotropic, semi-infinite, dry, elastic and cohesionless.
2. The face of the wall is contact with the backfill is vertical and inclined and rough.
3. The failure wedge acts as a rigid body and the stresses acting over it are uniform.

4. Failure is two dimensional, failure surface is planer and passes through the heel of the wall.
5. Point of application and line of action of the resultant thrust between the wall and soil is known.

NOTE

- In Rankine's theory, elemental failure is considered whereas is Coulomb's theory, wedge failure is considered.
- In Rankine's theory, the face of the wall in contact with the backfill is considered to be smooth and vertical whereas in Coulomb's theory, it is considered to be vertical or inclined and rough.
- In Coulomb's theory, the failure surface in passive state is considered planar but in actual it is circular. Hence, this theory is generally no suitable for passive state conditions.
- The main deficiency in Coulomb's theory is that in general it does not satisfy the static equilibrium conditions that occurs in nature.

11.9.1 Active State

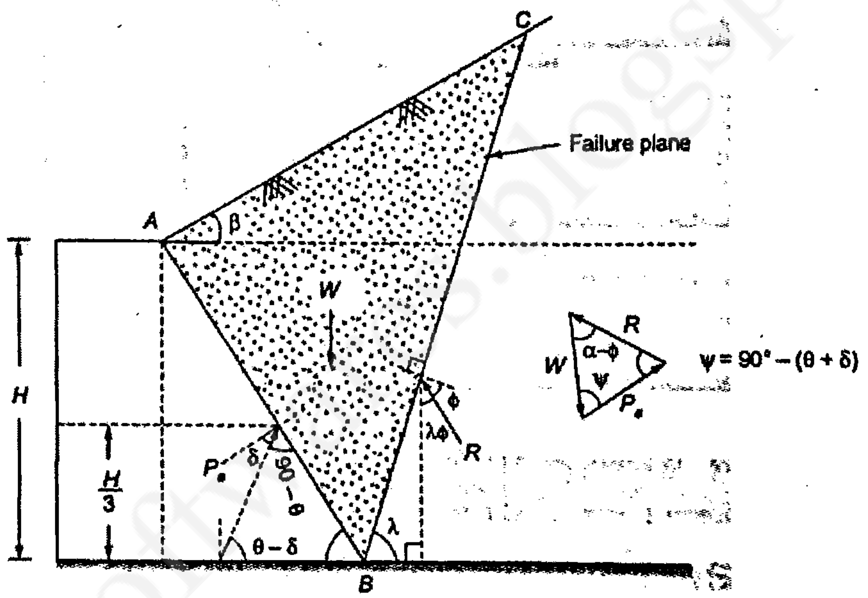


Fig. 11.36

θ = Angle made by inclined face of wall with vertical

δ = Angle of friction between soil and wall

λ = Angle made by failure plane with the horizontal

ϕ = Frictional angle of the soil

β = Surcharge angle

In this theory, equilibrium of failure wedge *ABC* is considered under three forces.

(i) Self-weight of the wedge *ABC* acting vertically downward.

(ii) Resultant thrust between the wall and soil acting at a downward angle of δ with the normal to the inclined face of the wall.

(iii) Resultant reaction *R*, between the two portions of the soil mass on either side of the failure plane acting at a downward angle of ϕ with the normal to failure plane.

Active earth pressure,

$$p_a = \frac{1}{2}k_a \gamma H^2$$

where
$$k_a = \left(\frac{\sec\theta\cos(\phi-\theta)}{\sqrt{\cos(\theta+\delta)}+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-\beta)}{\cos(\beta-\theta)}}}\right)^2$$

If $\beta = 0°$, $\theta = 0°$ and $\delta = \phi$

$$k_a = \left(\frac{1.\cos\phi}{\sqrt{\cos\phi}+\sqrt{\frac{\sin 2\phi \sin\phi}{1}}}\right)^2 = \frac{\cos\phi}{\left(1+\sqrt{2}\sin\phi\right)^2}$$

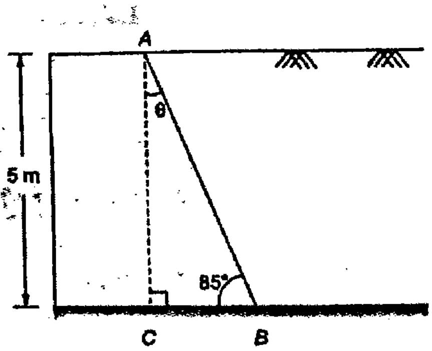
Line of action is at an angle δ with normal to inclined face of the wall.

The wall friction angle δ depends on the type of backfill and the type of wall.

S.No.	Type	Value of δ
1.	For smooth wall	$\frac{\phi}{3}$
2.	Ordinary retaining wall	$\frac{2\phi}{3}$
3.	Rough walls	$\frac{3\phi}{4}$

Example 11.17 A 5 m high retaining wall has a granular soil backfill with a level top. The retaining face makes an angle of 85° with the base. Soil parameters γ, ϕ and δ are 16 kN/m³, 35° and 10° respectively. Calculate active thrust per unit length of wall by using Coulomb's method.

Solution:



in ΔABC,

$$\theta = 180-(90+85°) = 5°$$

Here,
$$\beta = 0,\ \delta = 10°,\ \phi = 30°$$

$$k_a = \left(\frac{\sec\theta\cos(\phi-\theta)}{\sqrt{\cos(\theta+\delta)}+\sqrt{\frac{\sin(\delta+\theta)\sin(\phi-\beta)}{\cos(\beta-\theta)}}}\right)^2$$

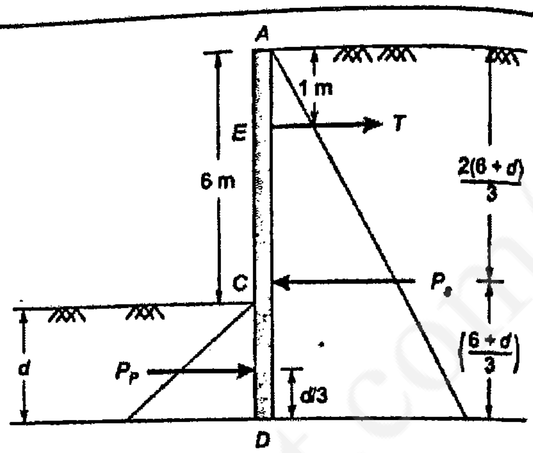
$$= \left(\frac{\sec 5°\cos(35°-5°)}{\sqrt{\cos(5°+10°)}+\sqrt{\frac{\sin(10°+35°)\sin(35°-0°)}{\cos(0°-5°)}}}\right)^2 = \left(\frac{\sec 5°\cos 30°}{\sqrt{\cos 15°}+\sqrt{\frac{\sin 45°\sin 35°}{\cos 5°}}}\right)^2$$

$$= \left(\frac{1.0038\times 0.8660}{.09828+0.6380}\right)^2 = (0.536)^2 = 0.288$$

Active thrust, $P_A = \frac{-1}{2}k_a\gamma H^2$

$$= \frac{1}{2}\times 0.288\times 16\times 5^2 = 57.6 \text{ kN/m}$$

Example 11.19 An anchored sheet pile is to support a mass of cohesionless soil up to a height of 6 m above ground level with horizontal anchor ties spaced at 1 m interval and located at 1.0 m below ground surface. If the unit weight of the soil is 21 kN/m³ and its angle of internal friction is 30°, determine the minimum depth of the sheet pile for stability.



Solution:

$$k_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

$$k_p = \frac{1}{k_a} = 3$$

$$P_a = \frac{1}{2}k_a\gamma(H+d)^2 = \frac{1}{2}\times\frac{1}{3}\times 21\times(6+d)^2 = 3.5(6+d)^2$$

$$P_p = \frac{1}{2}k_p\gamma d^2 = \frac{1}{2}\times 3\times 21\times d^2 = 31.5d^2$$

Taking moment about tie rod i.e., about E

$$P_a\left[\frac{2}{3}(6+d)-1\right]-P_p\left[(5+d)-\frac{d}{3}\right] = 0$$

$$3.5(6+d)^2\left[\frac{12+2d-3}{3}\right]-31.5d^2\left[\frac{15+2d}{3}\right] = 0$$

$$(6+d)^2(9+2d) = \frac{31.5d^2}{3.5}(15+2d)$$

$$(6+d)^2(9+2d) = 9d^2(15+2d)$$

By solving hit and trial, we get

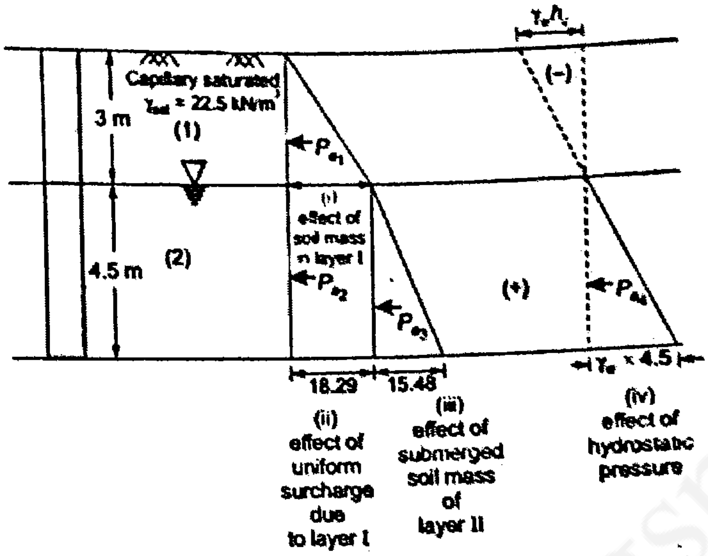
$$d = 2.3\text{ m}$$

Illustrative Examples

Example 11.20 A vertical wall H = 7.5 m is having cohesionless soil at the back having γ_{sat} = 22.5 kN/m³, ϕ = 35°. The water table behind the wall is at 3.0 m below top. The top 3 m soil is also saturated due to capillary moisture. Find the total thrust and point of application.

Solution:

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 35^\circ}{1+\sin 35^\circ} = \frac{0.4264}{1.5736} = 0.271$$



Soil Layer-I:

$$p_{a_1} = k_a \gamma_{sat} z$$

At point A ($z = 0$), $\quad p_{a_1} = 0$

At point B ($z = 3$ m), $\quad p_{a_1} = 0.271 \times 22.5 \times 3 = 18.29$ kN/m²

Active thrust contribution, $\quad P_{a_1} = \frac{1}{2} \times 18.29 \times 3 = 27.44$ kN/m

Line of action of P_{a_1}, $\quad \bar{h}_1 = \frac{3}{3} + 4.5 = 5.5$ m from base

NOTE: Tension due to capillary potential may be neglected as it does not play any role in development of cracks in soil.

Soil Layer-II:

(i) Effect of uniform surcharge due to top layer

$$p_{a_2} = k_a q = 0.271 \times \gamma \times 3 = 0.271 \times 22.5 \times 3 = 18.29 \text{ kN/m}^2 \text{ (constant)}$$

Active thrust contribution, $\quad P_{a_2} = 18.29 \times 4.5 = 82.30$ kN/m

Line of action of P_{a_2}, $\quad \bar{h}_2 = \frac{4.5}{2} = 2.25$ m from base

(iii) Effect of submerged soil mass

$$p_{a_3} = k_a \gamma' z$$

At point B($z = 0$), $\quad p_{a_3} = 0$

At point C($z = 4.5$ m) $\quad p_{a_3} = 0.271 \times (22.5 - 9.81) \times 4.5 = 15.48$ kN/m²

Active thrust contirbution, $P_{a_3} = \frac{1}{2} \times 15.48 \times 4.5 = 34.83$ kN/m

Line of action P_{a_3}, $\bar{H}_3 = \frac{4.5}{3} = 1.5$ m from base

(iii) Effect of hydrostatic pressure

$$p_{a_4} = \gamma_w z$$

At point B ($z = 0$), $p_{a_4} = 0$

At point C ($z = 4.5$ m), $p_{a_4} = 9.81 \times 4.5 = 44.15$ kN/m²

Thrust contribution, $P_{a_4} = \frac{1}{2} \times 44.15 \times 4.5 = 99.34$ kN/m

Line of action of P_{a_4}, $\bar{H}_4 = \frac{4.5}{3} = 1.5$ m from base

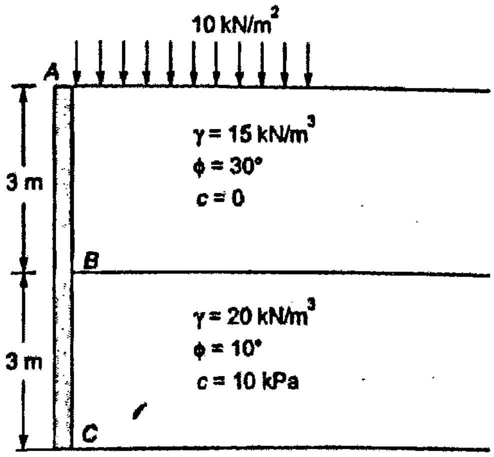
Total active thrust,

$$P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4}$$
$$= 27.44 + 82.30 + 34.83 + 99.34$$
$$= 243.91 \text{ kN/m}$$

Line of action of total thrust,

$$\bar{H} = \frac{27.44 \times 5.5 + 82.30 \times 2.25 + 34.83 \times 1.5 + 99.34 \times 1.5}{243.91}$$
$$= 2.203 \text{ m from base}$$

Example 11.21 A retaining wall with a stratified backfill and a surcharge load in shown in the following figure. Draw the earth pressure diagram. Also estimate the resultant thrust on the wall and its position.



Solution:

Top soil layer:

$$k_{a_1} = \frac{1-\sin\phi_1}{1+\sin\phi_1} = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

(i) Effect of uniform surcharge

$$p_{a_1} = k_{a_1} q = \frac{1}{3}\times 10 = 3.33 \text{ kN/m}^2 \text{ (constant)}$$

Thrust contribution, $P_{a_1} = 3.33 \times 3 = 10$ kN/m

Line of action of P_{a_1}, $\bar{h}_1 = \frac{3}{2} + 3 = 4.5$ m from base

(ii) Effect of ϕ-soil mass

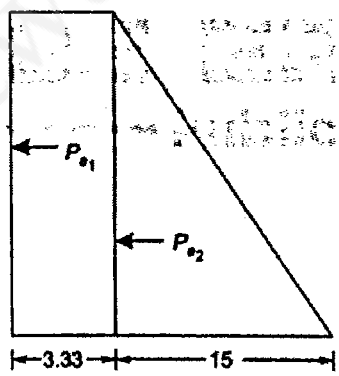
$$p_{a_2} = k_{a_1}\gamma_1 z$$

At point A ($z = 0$), $p_{a_2} = 0$

At point B ($z = 3$ m), $p_{a_2} = \frac{1}{3}\times 15\times 3 = 15 \text{ kN/m}^2$

Thrust contribution, $P_{a_2} = \frac{1}{2}\times 15\times 3 = 22.5$ kN/m

Line of action of P_{a_2}, $\bar{h}_2 = \frac{3}{3} + 3 = 4$ m from base



(i) Effect of uniform surcharge

(ii) Effect of ϕ-soil mass

Bottom Soil Layer:

$$k_{a_2} = \frac{1-\sin\phi_2}{1+\sin\phi_2} = \frac{1-\sin 10^\circ}{1+\sin 10^\circ} = 0.704$$

(i) Effect of equivalent surcharge

$$q' = q + \gamma_1 H_1 = 10 + 15 \times 3 = 55 \text{ kN/m}^2$$

$$p_{a_3} = k_{a_2} q' = 0.704 \times 55 = 38.72 \text{ kN/m}^2$$

Thrust contribution, $P_{a_3} = 38.72 \times 3 = 116.16$ kN/m

Line of action of P_{a_3}, $\bar{H}_3 = \frac{3}{2} = 1.5$ m from base

(ii) Effect of cohesion of soil

$$p_{a_4} = -2c\sqrt{k_{a_2}} = -2 \times 10 \times \sqrt{0.704} = -16.78 \text{ kN/m}^2$$

Thrust contribution, $P_{a_4} = -16.78 \times 3 = -50.34$ kN/m

Line of action of P_{a_4}, $\bar{H}_4 = \frac{3}{2} = 1.5$ m from base

(iii) Effect of internal friction of soil

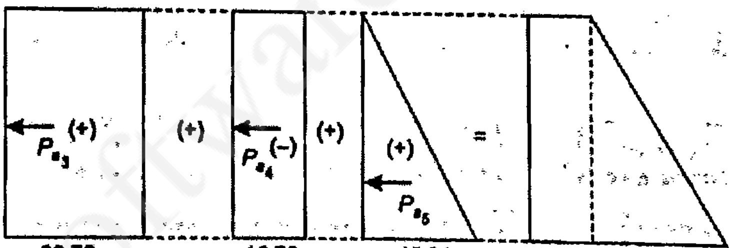
$$p_{a_5} = k_{a_2} \gamma_2 z$$

At point B ($z = 0$), $p_{a_5} = 0$

At point C ($z = 3$ m), $p_{a_5} = 0.704 \times 20 \times 3 = 42.24 \text{ kN/m}^2$

Thrust contribution, $P_{a_5} = \frac{1}{2} \times 42.24 \times 3 = 63.36$ kN/m

Line of action of P_{a_5}, $\bar{H}_5 = \frac{3}{3} = 1$ m from base



Total active thrust

$$P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4} + P_{a_5}$$
$$= 10 + 22.5 + 116.16 - 50.34 + 63.36$$
$$= 161.68 \text{ kN/m}$$

Line of action,

$$\bar{H} = \frac{\sum P_{a_i} \bar{H}_1}{\sum P_{a_i}}$$

$$= \frac{10 \times 4.5 + 22.5 \times 4 + 116.16 \times 1.5 - 50.34 \times 1.5 + 63.36 \times 1}{161.68}$$

$$= 1.8375 \text{ m from base}$$

If $D_f = 1.5$ m,

$$q_{ult} = \gamma D_f \left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 = 18 \times 1.5 \left(\frac{1+\sin 18°}{1-\sin 18°}\right)^2 = \mathbf{96.8\ kN/m^2}.$$

Example 14.3: Calculate the ultimate bearing capacity of a strip footing, 1 m wide, in a soil for $\gamma = 18$ kN/m^3, $c = 20$ kN/m^2, and $\phi = 20°$, at a depth of 1 m. Use Rankine's and Bell's approaches.

$\phi = 20°$

In Rankine's approach, cohesion is not considered.

$$q_{ult} = \frac{1}{2}\gamma b N_\gamma + \gamma D_f N_q$$

where
$$N_\gamma = \frac{1}{2}\sqrt{N_\phi}\,(N_\phi^2 - 1) \text{ and } N_q = N_\phi^2$$

$$N_\phi = \tan^2(45° + \phi/2) = \tan^2 55° = 2.04$$

$$N_\gamma = \frac{1}{2} \times \tan 55° (\tan^4 55° - 1) = 2.256$$

$$N_q = \tan^4 55° = 4.16$$

$\therefore$
$$q_{ult} = \frac{1}{2} \times 18 \times 1 \times 2.256 + 18 \times 1 \times 4.16 = \mathbf{95.2\ kN/m^2}$$

In Bell's approach, cohesion is also considered.

$$q_{ult} = cN_c + \frac{1}{2}\gamma b N_\gamma + \gamma D_f N_q$$

where $N_c = 2\sqrt{N_\phi}\,(N_\phi + 1)$, $N_\gamma = \frac{1}{2}\sqrt{N_\phi}\,(N_\phi^2 - 1)$, and $N_q = N_\phi^2$

$\therefore$
$$N_c = 2\tan 55° (\tan^2 55° + 1) = 8.682$$

$$N_\gamma = \frac{1}{2}\tan 55° (\tan^4 55° - 1) = 2.256$$

$$N_q = \tan^4 55° = 4.16$$

$\therefore$
$$q_{ult} = 20 \times 8.682 + \frac{1}{2} \times 18 \times 1 \times 2.256 + 18 \times 1 \times 4.16 = \mathbf{268.8\ kN/m^2}.$$

Example 14.4: A strip footing, 1.5 m wide, rests on the surface of a dry cohesionless soil having $\phi = 20°$ and $\gamma = 19$ kN/m^3. If the water table rises temporarily to the surface due to flooding, calculate the percentage reduction in the ultimate bearing capacity of the soil. Assume $N_\gamma = 5.0$. (S.V.U.—B.E., (Part-Time)—Apr., 1982)

$$\phi = 20° \quad N_\gamma = 5.0 \quad b = 1.5 \text{ m} \quad D_f = 0$$

Dry cohesionles soil, $\therefore$ $c = 0$

$$q_{ult} = cN_c + \frac{1}{2}\gamma b N_\gamma + \gamma D_f N_q = \frac{1}{2}\gamma b N_\gamma \text{ in this case.}$$

$$= \frac{1}{2} \times 19 \times 1.5 \times 5.0 = 71.3 \text{ kN/m}^2$$

If the water table rises temporarily to the surface due to flooding, reduction factors R_γ and R_q shall be applied as the maximum values for the N_γ and N_q terms respectively.

In this case, $R_\gamma = 0.5$ is applied for N_γ-term.

$\therefore \quad q_{ult} = \frac{1}{2}\gamma b N_\gamma . R_\gamma = \frac{1}{2} \times 19 \times 1.5 \times 5.0 \times 0.5 = 35.6 \text{ kN/m}^2$

The percentage reduction in the ultimate bearing capacity is thus 50 due to flooding and consequent complete submergence.

$\left(\text{Note: } \gamma_{sat} \text{ is assumed to be } \gamma \text{ itself here, and } \gamma' \approx \frac{1}{2}\gamma_{sat}\right)$

Example 14.5: A continuous footing of width 2.5 m rests 1.5 m below the ground surface in clay. The unconfined compressive strength of the clay is 150 kN/m^2. Calculate the ultimate bearing capacity of the footing. Assume unit weight of soil is 16 kN/m^3.

(S.V.U.—B.E., (R.R.)—May, 1969)

Continuous footing $b = 2.5$ m $D_f = 1.5$ m
Pure clay.

$\phi = 0°$ $\quad q_u = 150 \text{ kN/m}^2 \quad \gamma = 16 \text{ kN/m}^3$

$$c = \frac{q_u}{2} = 75 \text{ kN/m}^2$$

For $\phi = 0°$, Terzaghi's factors are: $N_\gamma = 0, N_q = 1$, and $N_c = 5.7$.

$$q_{ult} = cN_c + \frac{1}{2}\gamma b N_\gamma + \gamma D_f N_q = cN_c + \gamma D_f N_q, \text{ in this case.}$$

$$\therefore \quad q_{ult} = 5.7 \times 75 + 16 + 1.5 \times 1 = 451.5 \text{ kN/m}^2 \approx 450 \text{ kN/m}^2.$$

Example 14.6: Compute the safe bearing capacity of a continuous footing 1.8 m wide, and located at a depth of 1.2 m below ground level in a soil with unit weight $\gamma = 20$ kN/m^3, $c = 20$ kN/m^2, and $\phi = 20°$. Assume a factor of safety of 2.5. Terzaghi's bearing capacity factors for $\phi = 20°$ are $N_c = 17.7, N_q = 7.4$, and $N_\gamma = 5.0$, what is the permissible load per metre run of the footing ?

$b = 1.8$ m $\quad$ continuous footing $\quad D_f = 1.2$ m
$\gamma = 20$ kN/m^3 $\quad c = 20$ kN/m^2
$\phi = 20°$ $\quad N_c = 17.7$
$N_q = 7.4$ $\quad N_\gamma = 5.0$ $\quad \eta = 2.5$

$$q_{ult} = cN_c + \frac{1}{2}\gamma b N_\gamma + \gamma D_f N_q$$

$$= 20 \times 17.7 + \frac{1}{2} \times 20 \times 1.8 \times 5.0 + 20 \times 1.2 \times 7.4$$

$$= 621.6 \text{ kN/m}^2$$

$$q_{net\ ult} = q_{ult} - \gamma D_f = 621.6 - 20 \times 1.2 = 597.6 \text{ kN/m}^2$$

$$q_{net\ safe} = \frac{q_{net\ ult}}{\eta} = \frac{597.6}{2.5} = 239 \text{ kN/m}^2$$

$$q_{safe} = q_{net\ safe} + \gamma D_f = 239 + 20 \times 1.2 = 263 \text{ kN/m}^2$$

Permissible load per metre run of the wall = 263 × 1.8 kN = **473.5 kN.**

Example 14.7: What is the ultimate bearing capacity of a square footing resting on the surface of a saturated clay of unconfined compressive strength of 100 kN/m^2.

(S.V.U.—Four-year B. Tech.—Apr., 1983)

Square footing.
Saturated clay,

$\phi = 0° \quad D_f = 0.$

Terzaghi's factors for $\phi = 0°$ are : $N_c = 5.7$, $N_q = 1$, and $N_\gamma = 0$.

$$q_u = 100 \text{ kN/m}^2$$

$$\therefore \quad c = \frac{1}{2} q_u = 50 \text{ kN/m}^2$$

$$q_{ult} = 1.3\, cN_c = 1.3 \times 50 \times 5.7 = 370 \text{ kN/m}^2$$

$$\therefore \quad q_{ult} = 370 \text{ kN/m}^2.$$

Example 14.8: Determine the ultimate bearing capacity of a square footing of 1.5 m size, at a depth of 1.5 m, in a pure clay with an unconfined strength of 150 kN/m². $\phi = 0°$ and $\gamma = 17$ kN/m³.

(S.V.U.—Four-year B. Tech.,—Sept., 1983)

Square footing $b = 1.5$ m = 150 cm $D_f = 1.5$ m = 150 cm

Pure clay $\phi = 0°$, $q_u = 150$ kN/m², $\gamma = 17$ kN/m³

$$c = \frac{q_u}{2} = 75 \text{ kN/m}^2$$

Terzaghi's factors for $\phi = 0°$ are $N_c = 5.7$, $N_q = 1$, and $N_\gamma = 0$.

$$\therefore \quad q_{ult} = 1.3\, c\, N_c + \gamma D_f N_q + 0.4\, \gamma b\, N_\gamma = 1.3\, cN_c + \gamma D_f N_q, \text{ in this case}$$

$$q_{ult} = 1.3 \times 75 \times 5.7 + \frac{17 \times 150}{1000} \times 1 = 580 \text{ kN/m}^2$$

$$\therefore \quad q_{ult} = 580 \text{ kN/m}^2.$$

Example 14.9: A square footing, 1.8 m × 1.8 m, is placed over loose sand of density 16 kN/m³ and at a depth of 0.8 m. The angle of shearing resistance is 30°. $N_c = 30.14$, $N_q = 18.4$, and $N_\gamma = 15.1$. Determine the total load that can be carried by the footing.

(S.V.U.—Four-year B.Tech.,—Apr., 1983)

Square footing $b = 1.8$ m

$$\gamma = 16 \text{ kN/m}^3, \quad c = 0, \phi = 30°, \qquad D_f = 0.8 \text{ m}$$

$$N_c = 30.14, N_q = 18.4, \quad N_\gamma = 15.1$$

$$q_{ult} = 1.3\, cN_c + 0.4\gamma b\, N_\gamma + \gamma D_f N_q = 0.4\, \gamma b\, N_\gamma + \gamma D_f N_q, \text{ in this case}$$

$$\therefore \quad q_{ult} = 0.4 \times 16 \times 1.8 \times 15.1 + 16 \times 0.8 \times 18.4 = 174 + 236 = 410 \text{ kN/m}^2$$

The ultimate load that can be carried by the footing

$$= q_{ult} \times \text{Area} = 410 \times 1.8 \times 1.8 \text{ kN} = 1328.4 \text{ kN}.$$

Example 14.10: Compute the safe bearing capacity of a square footing 1.5 m × 1.5 m, located at a depth of 1 m below the ground level in a soil of average density 20 kN/m³. $\phi = 20°$, $N_c = 17.7$, $N_q = 7.4$, and $N_\gamma = 5.0$. Assume a suitable factor of safety and that the water table is very deep. Also compute the reduction in safe bearing capacity of the footing if the water table rises to the ground level.

(S.V.U.—B.Tech., (Part-time)—Sept., 1983)

$b = 1.5$ m Square footing $D_f = 1$ m

$\gamma = 20$ kN/m³ $\phi = 20°$ $N_c = 17.7$, $N_q = 7.4$, and $N_\gamma = 5.0$

Assume $c = 0$ and $\eta = 3$

$$q_{ult} = 1.3\, c\, N_c + 0.4\, \gamma\, b\, N_\gamma + \gamma D_f N_q = 0.4\, \gamma\, b\, N_\gamma + \gamma\, D_f N_q, \text{ in this case.}$$

$$= 0.4 \times 20 \times 1.5 \times 5.0 + 20 \times 1 \times 7.4 = 60 + 148 = 208 \text{ kN/m}^2$$

$$q_{net\,ult} = q_{ult} - \gamma D_f = 208 - 20 \times 1 = 188 \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{net\ ult}}{\eta} + \gamma D_f = \frac{188}{3} + 20 \times 1 = 83 \text{ kN/m}^2$$

If the water table rises to the ground level,

$$R_\gamma = 0.5 = R_q$$

$$\therefore \quad q_{ult} = 0.4\,\gamma^b N_\gamma \,.\, R_\gamma + \gamma D_f N_q \,.\, R_q$$

$$= 0.4 \times 20 \times 1.5 \times 5.0 \times 0.5 + 20 \times 1 \times 7.4 \times 0.5 = 30 + 74 = 104 \text{ kN/m}^2$$

$$q_{net\ ult} = q_{ult} - \gamma' D_f = 104 - 10 \times 1 = 94 \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{net\ ult}}{\eta} + \gamma' D_f = \frac{94}{3} + 10 \times 1 = 41 \text{ kN/m}^2$$

Percentage reduction in safe bearing capacity

$$= \frac{42}{83} \times 100 \approx 50.$$

Example 14.11: A foundation, 2.0 m square is installed 1.2 m below the surface of a uniform sandy gravel having a density of 19.2 kN/m³, above the water table and a submerged density of 10.1 kN/m³. The strength parameters with respect to effective stress are $c' = 0$ and $\phi' = 30°$. Find the gross ultimate bearing capacity for the following conditions:

(*i*) Water table is well below the base of the foundation (*i.e.*, the whole of the rupture zone is above the water table);

(*ii*) Water table rises to the level of the base of the foundation; and

(*iii*) the water table rises to ground level.

(For $\phi = 30°$, Terzaghi gives $N_q = 22$ and $N_\gamma = 20$)

(S.V.U.—B. Tech., (Part-time)—Sept., 1982)

Square $b = 2$ m $\quad D_f = 1.2$ m $\quad c' = 0 \quad \phi' = 30°$

$\gamma = 19.2$ kN/m³ $\quad \gamma' = 10.1$ kN/m³ $\quad N_q = 22 \quad N_\gamma = 20$

(*i*) Water table is well below the base of the foundation:

$$q_{ult} = 1.3\,c\,N_c + 0.4\,\gamma\,b\,N_\gamma + \gamma D_f N_q = 0.4\,\gamma\,b\,N_\gamma + \gamma D_f N_q, \text{ in this case.}$$

or $\quad q_{ult} = 0.4 \times 19.2 \times 2 \times 20 + 19.2 \times 1.2 \times 22 = 814 \text{ kN/m}^2$

(*ii*) Water table rises to the level of the base of the foundation:

$$q_{ult} = 0.4\,\gamma'\,b\,N_\gamma + \gamma\,D_f N_q$$

$$= 0.4 \times 10.1 \times 2 \times 20 + 19.2 \times 1.2 \times 22 = 668 \text{ kN/m}^2$$

(*iii*) Water table rises to the ground level:

$$q_{ult} = 0.4\,\gamma'\,bN_\gamma + \gamma' D_f N_q$$

$$= 0.4 \times 10.1 \times 2 \times 20 + 10.1 \times 1.2 \times 22 = 428 \text{ kN/m}^2$$

Thus, as the water table rises, there is about 20% to 50% decrease in the ultimate bearing capacity.

Example 14.12: The footing of a column is 2.25 m square and is founded at a depth of 1 m on a cohesive soil of unit weight 17.5 kN/m³. What is the safe load for this footing if cohesion = 30 kN/m²; angle of internal friction is zero and factor of safety is 3. Terzaghi's factors for $\phi = 0°$ are $N_c = 5.7, N_q = 1$, and $N_\gamma = 0$. (S.V.U.—B.E., (R.R.)—Feb., 1976)

Square $b = 2.25$ m $\quad D_f = 1$ m $\quad \gamma = 17.5$ kN/m³

$$q_{ult} = 1.3\,c\,N_c + 0.4\,\gamma b\,N_\gamma + \gamma D_f N_q = 1.3\,c\,N_c + \gamma D_f N_q, \text{ in this case since } N_\gamma = 0$$

$$\therefore \quad q_{ult} = 1.3 \times 30 \times 5.7 + 17.5 \times 1 \times 1 = 239.8 \text{ kN/m}^2$$

$$q_{net\,ult} = q_{ult} - \gamma D_f = 239.8 - 17.5 = 222.3 \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{net\,ult}}{\eta} + \gamma D_f = \frac{222.3}{3} + 17.5 = \mathbf{91.6 \text{ kN/m}^2}$$

Safe load on the footing:

$$= q_{safe} \times \text{Area} = 91.6 \times 2.25 \times 2.25 \approx \mathbf{463 \text{ kN}}.$$

Example 14.13: **What is the ultimate bearing capacity of a circular footing of 1 m diameter resting on the surface of a saturated clay of unconfined compression strength of 100 kN/m² ? What is the safe value if the factor of safety is 3?**

Diameter, $D = 1 \quad D_f = 0$

$$q_u = 1.3\,c\,N_c + 0.3\,\gamma D N_\gamma + \gamma D_f N_q$$

For saturated clay,

$$\phi = 0°$$

$$\therefore \quad N_c = 5.7 \quad N_q = 1 \quad N_\gamma = 0$$

Also $\quad c = \frac{1}{2} q_u = 50 \text{ kN/m}^2$

$$\therefore \quad q_{ult} = 1.3 \times 50 \times 5.7 = 370.5 \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{ult}}{\eta}, \text{ in this case,}$$

since $\quad N_q = 1$

$$\therefore \quad q_{safe} = \frac{370.5}{3} = \mathbf{123.5 \text{ kN/m}^2}.$$

Example 14.14: **A circular footing is resting on a stiff saturated clay with $q_u = 250$ kN/m². The depth of foundation is 2 m. Determine the diameter of the footing if the column load is 600 kN. Assume a factor of safety as 2.5. The bulk unit weight of soil is 20 kN/m³.**

(S.V.U.—Four-year B. Tech.—Dec., 1982)

Circular footing: $\phi = 0°, N_c = 5.7, N_q = 1, N_\gamma = 0$

$q_u = 250$ kN/m² $\quad c = \frac{1}{2} q_u = 125$ kN/m² $\quad D_f = 2$ m

Column load = 600 kN $\quad \eta = 2.5 \quad \gamma = 20$ kN/m³

$$q_{ult} = 1.3\,c\,N_c + 0.3\,\gamma D N_\gamma + \gamma D_f N_q = 1.3\,c\,N_c + \gamma D_f N_q, \text{ in this case.}$$

$$\therefore \quad q_{ult} = 1.3 \times 125 \times 5.7 + 20 \times 2 \times 1 = 966 \text{ kN/m}^2$$

$$q_{net\,ult} = q_{ult} - \gamma D_f = 966 - 20 \times 2 = 926 \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{net\,ult}}{\eta} + \gamma D_f = \frac{926}{2.5} + 20 \times 2 = 410 \text{ kN/m}^2$$

Safe load on the column

$$= q_{safe} \times \text{Area} = 600 \text{ kN}$$

$$\therefore \qquad 600 = \frac{410 \times \pi d^2}{4}$$

$$\therefore \qquad d = \sqrt{\frac{4 \times 600}{410}} \text{ m} = 1.365 \text{ m}$$

A diameter of 1.5 m may be adopted in this case.

Example 14.15: A column carries a load of 1000 kN. The soil is a dry sand weighing 19 kN/m³ and having an angle of internal friction of 40°. A minimum factor of safety of 2.5 is required and Terzaghi factors are required to be used. ($N_\gamma = 42$ and $N_q = 21$).

(*i*) Find the size of a square footing, if placed at the ground surface; and,

(*ii*) Find the size of a square footing required if it is placed at 1 m below ground surface with water table at ground surface. Assume $\gamma_{sat} = 21$ kN/m³.

(*i*) At ground surface:

$$\phi = 40° \text{ Dry sand}, N = 42\ N_q = 21$$

Let the size of the footing be b m.

$$q_{ult} = 0.4\ b \times 19 \times 42$$

Since $D_f = 0$, $\quad q_{net\ ult} = q_{ult}$

$$\therefore \qquad q_{safe} = q_{net\ ult} = \frac{q_{ult}}{\eta} = \frac{0.4\ b \times 19 \times 42}{2.5} = 128\ b \text{ kN/m}^2$$

$$Q_{safe} = 128\ b \times b^2 = 1000$$

$$\therefore \qquad b = \sqrt[3]{\frac{1000}{128}} \text{ m} = 2 \text{ m}$$

(*ii*) At 1 m below ground surface with water table at ground surface:

$$\gamma' = \gamma_{sat} - \gamma_w = (21 - 10) = 11 \text{ kN/m}^3$$

$$N = 42, N_\gamma = 21$$

$$q_{ult} = 0.4 \times \gamma b N_\gamma + \gamma D_f N_q$$

$$q_{ult} = 0.4\ b \times 11 \times 42 + 11 \times 1 \times 21 = (185\ b + 231) \text{ kN/m}^2$$

$$q_{net\ ult} = q_{ult} - \gamma D_f = 185\ b + 231 - 11 \times 1 = (185\ b + 220) \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{net\ ult}}{\eta} + \gamma D_f = \left[\frac{(185\ b + 200)}{2.5} + 11\right] \text{kN/m}^2$$

$$\therefore \qquad Q_{safe} = \left[\frac{185\ b + 220}{2.5} + 11\right] b^2 = 1000$$

Solving by trial and error, $\quad b = 2$ m.

$$b = 2 \text{ m.}$$

Example 14.16: What is the ultimate bearing capacity of a rectangular footing, 1 m × 2 m, on the surface of a saturated clay of unconfined compression strength of 100 kN/m² ?

Rectangular footing:

$$b = 1 \text{ m } L = 2 \text{ m } D_f = 0\ q_u = 100 \text{ kN/m}^2$$

$$c = \frac{1}{2} q_u = 50 \text{ kN/m}^2$$

Skempton's equation:

$$q_{net\ ult} = c \,.\, N_c, \text{ where } N_c = 5\left(1+0.2\frac{b}{L}\right)(1 + 0.2\, D_f/b) \text{ for } D_f/b \le 2.5)$$

$\therefore$ $$N_c = 5\left(1+0.2\times\frac{1}{2}\right) = 5.5, \text{ since } D_f = 0.$$

$\therefore$ $$q_{net\ ult} = 5.5 \times 50 = 275 \text{ kN/m}^2.$$

Since $D_f = 0$, $q_{ult} = q_{net\ ult} = 275 \text{ kN/m}^2$.

Example 14.17: **What is the safe bearing capacity of a rectangular footing, 1 m × 2 m, placed at a depth of 2 m in a saturated clay having unit weight of 20 kN/m³ and unconfined compression strength of 100 kN/m² ? Assume a factor of safety of 2.5.**

Rectangular footing:

$b = 1$ m $\quad L = 2$ m $\quad D_f = 2$ m $\quad q_u = 100 \text{ kN/m}^2 \quad \gamma = 20 \text{ kN/m}^3$

$D_f/b = \frac{2}{1} = 2 \quad b/L = \frac{1}{2} \quad c = \frac{1}{2} \quad q_u = 50 \text{ kN/m}^3$

Skempton's equation:

$$q_{net\ ult} = c \,.\, N_c, \text{ where } N_c = 5\left(1+0.2\frac{b}{L}\right)\left(1+0.2\frac{D_f}{b}\right) \text{ for } D_f/b \le 2.5$$

Since $D_f/b = 2 < 2.5$,

$$N_c = 5\left(1+0.2\times\frac{1}{2}\right)(1 + 0.2 \times 2/1) = 7.7$$

$\therefore$ $$q_{net\ ult} = 7.7 \times 50 = 385 \text{ kN/m}^2$$

$$q_{net\ safe} = \frac{q_{net\ ult}}{\eta} = \frac{385}{2.5} = 154 \text{ kN/m}^2$$

$$q_{safe} = q_{net\ safe} + \gamma D_f = 154 + 20 \times 2 = 194 \text{ kN/m}^2.$$

Example 14.18: **A steam turbine with base 6 m × 3.6 m weighs 10,000 kN. It is to be placed on a clay soil with $c = 135$ kN/m². Find the size of the foundation required if the factor of safety is to be 3. The foundation is to be 60 cm below ground surface.**

Skempton's equation:

$$q_{net\ ult} = 5c\left(1+0.2\frac{b}{L}\right)\left(1+0.2\frac{D_f}{b}\right) \text{ for } D_f/b \le 2.5.$$

$$D_f = 0.6 \text{ m}$$

For $\phi = 0°$, $N_\gamma = 0$ and $N_q = 1$ Assume $\gamma = 18$ kN/m³.

Adopt $b/L = 0.6$, same as that for the turbine base.

$$D_f/b = 0.6/b$$

Area, $$A = bL = \frac{b^2}{0.6} = \left(\frac{5b^2}{3}\right) \text{ m}^2$$

$\therefore$ $$q_{net\ ult} = 5 \times 135\,(1 + 0.2 \times 0.6)\left(1+\frac{0.2\times0.6}{b}\right) = 756\left(1+\frac{0.12}{b}\right) \text{ kN/m}^2$$

$$q_{safe} = \frac{q_{net\ ult}}{\eta} + \gamma D_f = \left[\frac{756\left(1+\frac{0.12}{b}\right)}{3} + 18 \times 0.6\right] \text{kN/m}^2$$

$$Q_{safe} = q_{safe} \times A = \frac{5b^2}{3}\left[756\,\frac{\left(1+\frac{0.12}{b}\right)}{3} + 10.8\right] \text{kN}$$

Equating Q_{safe} to 10,000, we have

$$420\,b^2\left(1+\frac{0.12}{b}\right) + 18\,b^2 = 10{,}000$$

Solving for b,

$b = 4.72$ m, say 4.80 m. ($D_f/b < 2.5$ is satisfied)

$L = 4.8/0.6 = 8.0$ m

Hence, the size of the foundation required is 4.8 m × 8.0 m.

Example 14.19: Calculate the ultimate bearing capacity, according to the Brinch Hansen's method, of a rectangular footing 2 m × 3 m, at a depth of 1 m in a soil for which $\gamma = 18$ kN/m^2, $c = 20$ kN/m^2, and $\phi = 20°$. The ground water table is lower than 3 m from the surface. The total vertical load is 1350 kN and the total horizontal load is 75 kN at the base of the footing. Hansen's factors for $\phi = 20°$ are $N_c = 14.83$, $N_q = 6.40$, and $N_\gamma = 3.54$. Determine also the factor safety.

Rectangular footing:

$\phi = 20°$, $N_c = 14.83$, $N_q = 6.40$, $N_\gamma = 3.54$, $D_f = 1$ m
(Hansen's factors)

$c = 20$ kN/m^2 $q = \gamma D_f = 18 \times 1 = 18$ kN/m^2 $\gamma = 18$ kN/m^3

$b = 2$ m $L = 2$ m $A = 6$ m^2 $H = 75$ kN $V = 1350$ kN

Hansen's formula:

$$q_{ult} = c\,N_c s_c d_c i_c + qN_q s_q d_q i_q + \frac{1}{2}\gamma b\,N_\gamma s_\gamma d_\gamma i_\gamma$$

Shape factors:

$$s_c = 1 + 0.2\,b/L = 1 + 0.2 \times \frac{2}{3} = 1.133$$

$$s_q = 1 + 0.2\,b/L = 1.133$$

$$s_\gamma = 1 - 0.4\,b/L = 1 - 0.4 \times \frac{2}{3} = 0.733$$

Depth factors:

$$d_c = 1 + 0.35\,D_f/b = 1 + 0.35 \times \frac{1}{2} = 1.175$$

$$d_q = 1 + 0.35\,D_f/b = 1.175$$

$$d_\gamma = 1.0$$

$d_\gamma = 1.0$

Inclination factors:

$i_q = i_\gamma = 1$ for purely vertical loading.

(*i*) G.W.L. at 8 m below natural ground level:

$\gamma = \gamma_d$ in both terms.

$$\therefore \quad q_{ult} = 15.3 \times 1.5 \times 18.40 \times 1.133 \times 1.263 \times 1 + \frac{1}{2} \times 15.3 \times 2 \times 18.08 \times 0.733 \times 1 \times 1$$

$$= 604.27 + 202.76 = \mathbf{807.03\ kN/m^2}$$

(*ii*) G.W.L. at 1.5 m below natural ground level:

$\gamma = \gamma_d$ in the first term and $\gamma = \gamma'$ in the second term.

$$\therefore \quad q_{ult} = 15.3 \times 1.5 \times 18.40 \times 1.133 \times 1.263 \times 1 + \frac{1}{2} \times 9.40 \times 2 \times 18.08 \times 0.733 \times 1 \times 1$$

$$= 604.27 + 124.58 = \mathbf{728.85\ kN/m^2}$$

Thus, there is about 10% decrease in bearing capacity as the water table rises to the level of the base of the footing.

Example 14.21: A foundation 2 m × 3 m is resting at a depth of 1 m below the ground surface. The soil has a unit cohesion of 10 kN/m², angle of shearing resistance of 30° and unit weight of 20 kN/m³. Find the ultimate bearing capacity using Balla's method.

Balla's equation:

$$q_{ult} = cN_c + qN_q + \frac{1}{2}\gamma b N_\gamma$$

$$D_f/b = \frac{1}{2} = 0.5$$

$$\frac{c}{\gamma b} = \frac{10}{20 \times 2} = 0.25$$

For $\phi = 30°$ and $\frac{c}{\gamma b} = 0.25$

$\rho = 1.9$ from the charts for $D_f/b = 0.5$

For $\phi = 30°$ and $\rho = 1.9$, from the relevant charts, Balla's factors are:

$N_c = 37$

$N_q = 25$

$N_\gamma = 64$

Hence, $$q_{ult} = 10 \times 37 + 20 \times 1 \times 25 + \frac{1}{2} \times 20 \times 2 \times 64$$

$$= 370 + 500 + 1280 = \mathbf{2{,}150\ kN/m^2}.$$

(**Note.** Strictly speaking, Balla's method is applicable only for continuous footings).

Example 14.22: A plate load test was conducted on a uniform deposit of sand and the following data were obtained:

Pressure kN/m²	50	100	200	300	400	500	600
Settlement mm	1.5	2.0	4.0	7.5	12.5	20.0	40.0

The size of the plate was 750 mm × 750 mm and that of the pit 3.75 m × 3.75 m × 1.5 m.

(*i*) Plot the pressure-settlement curve and determine the failure stress.

(*ii*) A square footing, 2m × 2 m, is to be founded at 1.5 m depth in this soil.

Assuming the factor of safety against shear failure as 3 and the maximum permissible settlement as 40 mm, determine the allowable bearing pressure.

(*iii*) Design of footing for a load of 2,000 kN, if the water table is at a great depth.

(*i*) The pressure-settlement curve is shown in Fig. 14.23

The failure point is obtained as the point corresponding to the intersection of the initial and final tangents. In this case, the failure stress is 500 kN/m².

$\therefore \qquad q_{ult} = 500 \text{ kN/m}^2$

(*ii*) The value of q_{ult} here is given by $\frac{1}{2}\gamma b_p N_\gamma$.

b_p, the size of test plate = 0.75 m

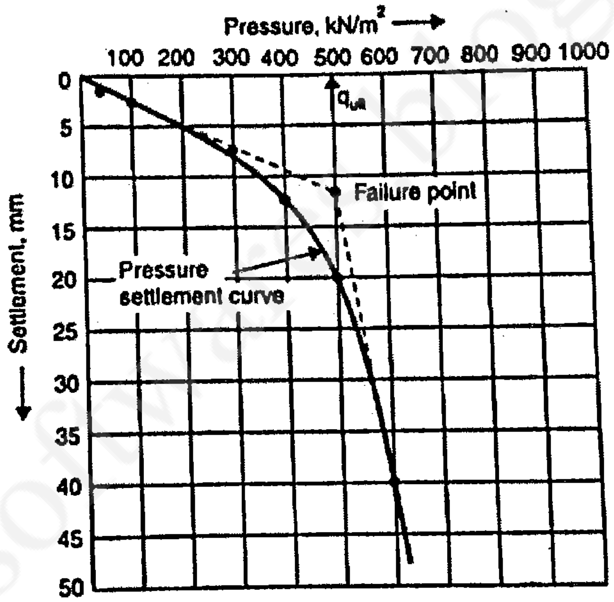


Fig. 14.23 Pressure-settlement curve (Ex. 14.22)

Assuming γ = 20 kN/m³,

$$500 = \frac{1}{2} \times 20 \times 0.75\, N_\gamma$$

$$\therefore \qquad N_\gamma = 500/7.5 \approx 6.7$$

$$\phi = 38°$$

$$\therefore \qquad N_q \approx 50 \text{ from Terzaghi's charts.}$$

For square footing of size 2 m and D_f = 1.5 m,

$$q_{net\ ult} = 0.4\,\gamma\, b\, N_\gamma + \gamma D_f (N_q - 1)$$

$$= 0.4 \times 20 \times 2 \times 67 + 20 \times 1.5 \times 49 = 2{,}542 \text{ kN/m}^2$$

$$q_{safe} = \frac{2,542}{3} \approx 847 \text{ kN/m}^2 \text{ (for failure against shear)}$$

From settlement consideration,

$$\frac{S_p}{S} = \left[b_p \frac{(b+0.30)}{b(b_p+0.30)}\right]^2 \quad S_p = S\left[\frac{b_p(b+0.30)}{b(b_p+0.30)}\right]^2$$

$$= 40\left[\frac{0.75(2+0.30)}{2.00(0.75+0.30)}\right]^2 = 40\left(\frac{0.75\times 2.3}{2\times 1.05}\right)^2 \text{ mm} = 27 \text{ mm}$$

Pressure for a settlement of 27 mm for the plate (from Fig. 14.23) = 550 kN/m^2.

Allowable bearing pressure is the smaller of the values from the two criteria = 550 kN/m^2.

(*iii*) Design load = 2,000 kN

From Part (*ii*), it is known that a 2 m square footing can carry a load of 2 × 2 × 550 = 2,200 kN.

Therefore, a 2 m square footing placed at a depth of 1.5 m is adequate for the design load.

Example 14.23: A loading test was conducted with a 300 mm square plate at depth of 1 m below the ground surface in pure clay deposit. The water table is located at a depth of 4 m below the ground level. Failure occurred at a load of 45 kN.

What is the safe bearing capacity of a 1.5 m wide strip footing at 1.5 m depth in the same soil? Assume γ = 18 kN/m^3 above the water table and a factor of safety of 2.5.

The water table does not affect the bearing capacity in both cases.

Test plate:

$$q_{ult} = \frac{\text{Failure load}}{\text{Area of plate}} = \frac{45}{0.3\times 0.3} = 500 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

For $\phi = 0°$, Terzaghi's factors are $N_c = 5.7$, $N_q = 1$, and $N_\gamma = 0$

$$\therefore \quad q_{ult} = 1.3\, c\, N_c + \gamma D_f N_q$$

$$= 1.3 \times 5.7c + 18 \times 1.0$$

$$= 7.4\, c + 18 \text{ (kN/m}^2\text{, if } c \text{ is expressed in kN/m}^2\text{)}$$

$$\therefore \quad 7.4\, c + 18 = 500$$

$$\therefore \quad c = \frac{482}{7.4} \approx 65 \text{ kN/m}^2$$

Strip footing:

$$q_{ult} = c N_c + \gamma D_f N_q = 5.7\, c + \gamma D_f \text{ in this case.}$$

$$q_{net\ ult} = q_{ult} - \gamma D_f = 5.7\, c$$

$$\therefore \quad q_{net\ ult} = 5.7 \times 65 = 370.5 \text{ kN/m}^2$$

$$q_{net\ safe} = \frac{370.5}{2.5} \approx 148 \text{ kN/m}^2$$

$$q_{safe} = q_{net\ safe} + \gamma D_f = 148 + 18 \times 1.5 = 175 \text{ kN/m}^2$$

However, the net safe bearing capacity is used in the design of footings in clay.

Example 14.24: A continuous wall footing, 1 m wide, rests on sand, the water table lying at a great depth. The corrected N-value for the sand is 10. Determine the load which the footing can support if the factor of safety against bearing capacity failure is 3 and the settlement is not to exceed 40 mm.

Continuous footing:

$$b = 1 \text{ m}$$

Let us assume a minimum depth, D_f, of 0.5 m.

Assume $\gamma = 18$ kN/m³

For $N = 10$, $\phi = 30°$ (Fig. 14.18 or 14.19, after Peck, Hanson and Thornburn)

Assuming general shear failure,

for $\phi = 30°$, Terzaghi's factors are: $N_q = 22.5$ and $N_\gamma = 19.7$

$$\therefore \quad q_{\text{net ult}} = \frac{1}{2}\gamma b N_\gamma + \gamma D_f (N_q - 1)$$

$$= \frac{1}{2} \times 18 \times 1 \times 19.7 + 18 \times 0.5 \times 21.5 \approx 370 \text{ kN/m}^2$$

$$q_{\text{net safe}} = \frac{q_{\text{net ult}}}{\eta} = \frac{370}{3} \approx 123 \text{ kN/m}^2$$

For $N = 10$ and $b = 1$m, and for a permissible settlement of 40 mm, the allowable soil pressure = 120 kN/m² (From Fig. 14.22, after Peck, Hanson and Thornburn)

$\therefore$ Allowable bearing pressure ≈ 120 kN/m²

(*i.e.*, shear failure governs the design).

Load which the footing can support = 120 × 1 kN/m = **120 kN/metre run.**

Example 14.25: What load can a 2 m square column carry in a dense sand ($\gamma = 20$ kN/m³ and $\phi = 36°$) at a depth of 1 m, if the settlement is not to exceed 30 mm ? Assume a factor of safety of 3 against shear and that the ground water table is at a great depth.

$\gamma = 20$ kN/m³ $D_f = 1$ m $b = 2$ m (Square footing) $\phi = 36°$

For $\phi = 36$, $N = 30$ (from Fig. 14.18, after Peck, Hanson and Thornburn)

For $N = 30$, $b = 2$ m permissible settlement = 40 mm,

Allowable soil pressure = 500 kN/m². (from Fig. 14.21 after Terzaghi and Peck)

For a permissible settlement of 30 mm,

$$\text{Allowable pressure} = \frac{500 \times 30}{40} = 375 \text{ kN/m}^2$$

For $\phi = 36°$, $N_q = 43$ and $N_\gamma = 46$

$$\therefore \quad q_{\text{net ult}} = 0.4\,\gamma\, b\, N_\gamma + \gamma D_f (N_q - 1)$$

$$= 0.4 \times 20 \times 2 \times 46 + 20 \times 1 \times 42 = 736 + 840 = 1576 \text{ kN/m}^2$$

$$q_{\text{safe}} = \frac{1576}{3} = 525 \text{ kN/m}^2$$

$\therefore$ Settlement governs the design, and

$$q_{\text{allowable}} = 375 \text{ kN/m}^2$$

Safe load for the column = 375 × 2 × 2 = 1500 kN.

Example 14.26: **A footing, 2 m square, is founded at a depth of 1.5 m in a sand deposit, for which the corrected value of N is 27. The water table is at a depth of 2 m from the surface. Determine the net allowable bearing pressure, if the permissible settlement is 40 mm and a factor of safety of 3 is desired against shear failure.**

Settlement criterion:

$$N = 27 \quad b = 2 \text{ m} \quad D_f = 1.5 \text{ m}$$

Using Teng's equation for the graphical relationship of Terzaghi and Peck (Fig. 14.20) for a settlement of 25 mm,

$$q_{na} = 34.3\,(N-3)\left(\frac{b+0.3}{2b}\right)^2 R_\gamma \,.\, R_d$$

q_{na} is in kN/m² b = Width in m

R_γ is the correction factor for the location water table.

R_d is the depth factor.

$$z_\gamma = (2-1.5) = 0.5 \text{ m} \quad \therefore \quad z_q > D_f$$

$$\therefore \quad \frac{z_\gamma}{b} = 0.5/2 = 0.25$$

$$R_q = 1.0 \text{ (limiting value)}$$

$$R_\gamma = 0.5\left(1+\frac{z_\gamma}{b}\right) = 0.5(1+0.25) = 0.625$$

$$R_d = 1 + \frac{D_f}{b} = 1 + \frac{15}{2} = 1.75$$

$$\therefore \quad q_{na} = 34.3 \times 24 \times \left(\frac{2.3}{4}\right)^2 \times 0.625 \times 1.75 \text{ kN/m}^2 \approx 298 \text{ kN/m}^2.$$

Since this is for a settlement of 25 mm,

$$q_{na}, \text{ for settlement of 40 mm} = 298 \times \frac{40}{25} \approx 476 \text{ kN/m}^2.$$

Shear failure criterion:

For a factor of safety of 3, Teng's equation for Terzaghi's bearing capacity equation is:

$$q_{ns} = \frac{1}{18}\,[2N^2\, b\, R_\gamma + 6\,(100 + N^2)\, D_f R_q],$$

neglecting (2/3) γD_f (for square footing)

$$= \frac{1}{18}\,[2 \times 27^2 \times 2 \times 0.625 + 6(100 + 27^2)1.5 \times 1.0] \approx 516 \text{ kN/m}^2$$

Hence settlement governs the design and the allowable bearing pressure is 476 kN/m². (**Note:** This is conservative and if Bowles' recommendation is considered, it can be enhanced by 50%; in this case shear failure governs the design, and q_{safe} will be 516 kN/m²).

***Example* 14.27:** Two load tests were conducted at a site—one with a 0.5 m square test plate and the other with a 1.0 m square test plate. For a settlement of 25 mm, the loads were found to be 60 kN and 180 kN, respectively in the two tests. Determine the allowable bearing pressure of the sand and the load which a square footing, 2 m × 2 m, can carry with the settlement not exceeding 25 mm.

$$q_{ult} = mx + \sigma$$

where x = Perimeter-area ratio, P/A

First test | *Second test*

$$x_1 = \frac{P_1}{A_1} = \frac{4 \times 0.5}{0.5 \times 0.5} = 8 \text{ m}^{-1} \qquad x_2 = \frac{P_2}{A_2} = \frac{4 \times 1}{1 \times 1} = 4 \text{ m}^{-1}$$

$$q_1 = \frac{60}{0.5 \times 0.5} = 240 \text{ kN/m}^2 \qquad q_2 = \frac{180}{1 \times 1} = 180 \text{ kN/m}^2$$

$\therefore$ $240 = 8\text{ m} + \sigma$ ....(1) $180 = 4\text{ m} + \sigma$ ...(2)

Solving Eqs. (1) and (2) simultaneously,

$$m = 15 \text{ (kN/m)} \quad \text{and} \quad \sigma = 120 \text{ (kN/m}^2\text{)}$$

Prototype footing:

$$x = P/A = \frac{4 \times 2}{2 \times 2} = 2 \text{ m}^{-1}$$

$$\therefore \quad q = mx + \sigma$$
$$= 15 \times 2 + 120 = 150 \text{ kN/m}^2$$

This is the allowable bearing pressure for a settlement of 25 mm. Load which the footing can carry,

$$Q_s = q_s \times \text{Area} = 150 \times 2 \times 2 = 600 \text{ kN.}$$

SUMMARY OF MAIN POINTS

1. The load-carrying capacity of a foundation to transmit loads from the structure to the foundation soil is termed its 'bearing capacity'. The criteria for the determination of the bearing capacity are avoidance of the risk of shear failure of the soil and of detrimental settlements of the foundation.

   Safe bearing capacity is the ultimate value divided by a suitable factor of safety; the allowable bearing pressure is the smaller safe capacity from the two criteria of shear failure and settlement.
2. The factors on which the bearing capacity depends are the size, shape and depth of the foundation and soil characteristics, including the location of the GWT relative to the foundation.
3. The methods of determination of bearing capacity are selection from building codes, analytical methods, plate load tests, penetration tests, model tests, and laboratory tests.
4. Of the analytical methods, Schleicher's is based on the theory of elasticity, Rankine's and Bell's are based on Rankine's classical theory of earth pressure, Fellenius', Prandtl's, Terzaghi's, Meyerhof's, Skempton's and Brinch Hansen's methods are based on the theory of plasticity.

EXAMPLES

Example 15.1 Determine the ultimate bearing capacity of a strip footing, 1.5 m wide, with its base at a depth of 1 m, resting on a dry sand stratum.

Take $\gamma_d = 17$ kN/m^3, $\phi' = 38°$ and $c' = 0$. Use Terzaghi's theory.

Solution:

$\phi' > 36°$. Hence general shear failure will occur. The ultimate bearing capacity of a strip footing in general shear failure is given by

$$q_u = cN_c + qN_q + 0.5\,\gamma B N_\gamma$$

For a free-draining soil like sand, the bearing capacity analysis is carried out in terms of effective stresses. Since $c' = 0$,

$$q_u = qN_q + 0.5\,\gamma B N_\gamma$$

For $\phi' = 38°$, Fig. 15.9 gives $N_q = 60$ and $N_\gamma = 75$

$$q = \gamma_d D_f = 17 \times 1 = 17 \text{ kN/m}^2.$$

$$\therefore \quad q_u = 17 \times 60 + 0.5 \times 17 \times 1.5 \times 75$$

$$= 1976 \text{ kN/m}^2$$

Example 15.2 In Ex. 15.1, determine the ultimate bearing capacity of the footing using the bearing capacity factors recommended by Meyerhof, Hansen and IS: 6403 (1981). Ignore embedment effect.

Solution:

For $\phi' = 38°$, from Tables 15.2, 15.4 and 15.6

$$N_q = 48.9;\ N_\gamma = 64.0 \text{ (Meyerhof)}$$

$$N_q = 48.9;\ N_\gamma = 56.2 \text{ (Hansen)}$$

$$N_q = 48.9;\ N_\gamma = 78.0 \text{ (IS: 6403-1981)}$$

Substituting these in the bearing capacity equation,

$$q_u = 17 \times 48.9 + 0.5 \times 17 \times 1.5 \times 64 = 1647 \text{ kN/m}^2 \text{ (Meyerhof)}$$

$$q_u = 17 \times 48.9 + 0.5 \times 17 \times 1.5 \times 56.2 = 1545 \text{ kN/m}^2 \text{ (Hansen)}$$

$$q_u = 17 \times 48.9 + 0.5 \times 17 \times 1.5 \times 78.0 = 1826 \text{ kN/m}^2 \text{ (IS: 6403)}$$

Example 15.3 If the footing in Ex. 15.1 is resting on a sand stratum with $\phi' = 32°$, determine the ultimate bearing capacity using Terzaghi theory, assuming all other values to be the same.

Solution:

Since ϕ' lies between 29° and 36°, the bearing capacity factors are obtained by interpolation between the local and general shear failure conditions.

By referring to both sets of curves for $\phi' = 32°$ in Fig. 15.9,

$$N_q = 25; N'_q = 10. \text{ Hence actual}$$

$$N_q = 10 + 15\left(\frac{32-29}{36-29}\right) = 16.4$$

$$N_\gamma = 28; N'_\gamma = 6. \text{ Hence actual}$$

$$N_\gamma = 6 + 22\left(\frac{32-29}{36-29}\right) = 15.4$$

Hence, $$q_u = 17 \times 16.4 + 0.5 \times 17 \times 1.5 \times 15.4$$

$$= 475 \text{ kN/m}^2$$

Example 15.4 Determine the ultimate bearing capacity of the footing in Ex. 15.1 if the ground water table is located (a) at a depth of 0.5 m below the ground surface, (b) at a depth of 0.5 m below the base of the footing. $\gamma_{sat} = 20 \text{ kN/m}^3$. Use Terzaghi theory.

Solution:

$q_u = qN_q + 0.5\,\gamma BN_\gamma$

where q is the effective surcharge and γ is the effective unit weight of the soil beneath the footing.

(a) q is calculated by assuming the unit weight to the top 0.5 m soil to be unchanged, that is, 17 kN/m^3 while the remaining 0.5 m is submerged, with a submerged unit weight equal to 10 kN/m^3 ($\gamma' = \gamma_{sat} - \gamma_w$).

Thus $$q = 0.5 \times 17 + 0.5 \times 10 = 13.5 \text{ kN/m}^2$$

In the second part, $$\gamma = \gamma' = 10 \text{ kN/m}^3$$

$$q_u = 13.5 \times 60 + 0.5 \times 10 \times 1.5 \times 75$$

$$= 1372.5 \text{ kN/m}^2$$

compared with $q_u = 1976 \text{ kN/m}^2$ when there was no effect of water table.

(b) For this case, $q = 17 \text{ kN/m}^2$

In the term $0.5\,\gamma BN_\gamma$, γ is given by Eq. 15.30

$$\gamma = \gamma' + \left(\frac{D'_w}{B}\right)(\gamma_t - \gamma')$$

$$D'_w = 0.5 \text{ m}, \gamma_t = 17 \text{ kN/m}^3 \text{ and } \gamma' = 10 \text{ kN/m}^3$$

$$\gamma = 10 + \frac{0.5}{1.5}(17 - 10) = 12.3 \text{ kN/m}^3$$

Hence, $$q_u = 17 \times 60 + 0.5 \times 12.3 \times 1.5 \times 75$$

$$= 1711.9 \text{ kN/m}^2$$

Example 15.5 Determine the ultimate bearing capacity of a strip footing 2 m in width, with its base at a depth of 1.5 m below ground surface and resting on a saturated clay soil with the following properties:

$$\gamma_{sat} = 20 \text{ kN/m}^3;\ c_u = 40 \text{ kN/m}^2;\ \phi_u = 0$$
$$c' = 10 \text{ kN/m}^2;\ \phi' = 20°$$

The natural water table is at 1 m depth below ground level. Ignore depth factors.

Solution:

The ultimate bearing capacity for the undrained and drained loading conditions can be determined by using the total stress and effective stress shear strength parameters respectively, for the clay soil.

(a) Undrained loading condition

In Eq. 15.19, $\phi_u = 0$

Hence $q_u = c_u N_c + q$ since $N_c = 1$ and $N_\gamma = 0$ in the Terzaghi, Meyerhof, Hansen and IS: 6403-1981 recommendations.

$$q = 20 \times 1 + 10 \times 0.5 = 25 \text{ kN/m}^2;\ c_u = 40 \text{ kN/m}^2$$
$$q_u = 40 \times 5.7 + 25 = 253 \text{ kN/m}^2 \text{ (Terzaghi)}$$
$$q_u = 40 \times 5.14 + 25 = 231 \text{ kN/m}^2 \text{ (Meyerhof, Hansen and IS: 6403-1981)}$$

(b) Drained loading condition

$$q_u = c' N_c + q N_q + 0.\gamma B N_\gamma$$
$$q = 25 \text{ kN/m}^2;\ \gamma = \gamma' = 10 \text{ kN/m}^2$$

Appropriate N_c, N_q, N_γ values are substituted for the different cases for $\phi' = 20°$

$$q_u = 10 \times 17.7 + 25 \times 7.4 + 0.5 \times 10 \times 2 \times 5.0 = 412 \text{ kN/m}^2 \text{ (Terzaghi)}$$
$$q_u = 10 \times 14.8 + 25 \times 6.4 + 0.5 \times 10 \times 2 \times 2.9 = 337 \text{ kN/m}^2 \text{ (Meyerhof and Hansen)}$$
$$q_u = 10 \times 14.8 + 25 \times 6.4 + 0.5 \times 10 \times 2 \times 5.4 = 362 \text{ kN/m}^2 \text{ (IS: 6403 - 1981)}$$

It can be seen that the ultimate bearing capacity under drained loading condition is higher than under undrained loading condition. That is why the bearing capacity analysis for a clay soil is carried out invariably under undrained loading condition.

Example 15.6 In Ex. 15.2, if the effect of embedment is accounted for, what will be the changes in the values of ultimate bearing capacity?

Solution:

$$q_u = qN_q d_q + 0.5 \gamma B N_\gamma d_\gamma$$

From Table 15.3,

$$d_q = d_\gamma = 1 + 0.1 \frac{D}{B} \tan(45 + \phi/2) = 1 + 0.1 \frac{1}{1.5} \tan\left(45 + \frac{38}{2}\right)$$
$$= 1.137 \text{ (Meyerhof)}$$

From Table 15.5,

$$d_q = 1 + 2\tan\phi\,(1-\sin\phi)^2\left(\frac{D}{B}\right)$$

$$= 1 + 2\tan 38°\,(1-\sin 38°)^2\left(\frac{1.5}{1}\right)$$

$$= 1.154 \text{ (Hansen)}$$

$$d_\gamma = 1 \text{ (Hansen)}$$

From Table 15.7,

$$d_q = d_\gamma = 1 + 0.1\frac{D}{B}\tan(45° + \phi/2)$$

$$= 1.137 \text{ (IS : 6403–1981)}$$

Using these depth factors,

$$q_u = 1873 \text{ kN/m}^2 \text{ (Meyerhof)}$$

$$q_u = 1676 \text{ kN/m}^2 \text{ (Hansen)}$$

$$q_u = 2076 \text{ kN/m}^2 \text{ (IS: 6403-1981)}$$

Example 15.7 Calculate the net ultimate bearing capacity of a rectangular footing 2m × 4m in plan, founded at a depth of 1.5 m below the ground surface. The load on the footing acts at an angle of 15° to the vertical and is eccentric in the direction of width by 15 cm. The saturated unit weight of the soil is 18 kN/m^3. The rate of loading is slow and hence the effective stress shear strength parameters can be used in the analysis. $c' = 15$ kN/m^2 and $\phi' = 25°$. Natural water table is at a depth of 2 m below the ground surface. Use IS: 6403-1981 recommendations.

Solution

Form Eq. 15.54,

$$q_v = cN_c s_c d_c i_c + q(N_q - 1)s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma W'$$

$$c = c' = 15 \text{ kN/m}^2$$

$$\phi = \phi' = 25°$$

From Tables 15.2 and 15.6, for $\phi = 25°$,

$$N_c = 20.7;\; N_q = 10.7 \text{ and } N_\gamma = 10.9$$

$$\gamma = \gamma_{sat} = 18 \text{ kN/m}^3$$

$$q = 18 \times 1.5 = 27 \text{ kN/m}^2$$

For $$\frac{D'_w}{B} = 0.25 \quad (D'_w = 2.0 - 1.5 = 0.5 \text{ m}),\; W' = 0.625 \text{ (Fig. 15.10)}$$

$$e_x = 0.15 \text{ m; effective width } B' = B - 2e_x = 2 - 0.3 = 1.7 \text{ m}$$

From Table 15.7,

$$s_c = s_q = 1 + 0.2\frac{B'}{L} = 1 + 0.2\,(\frac{1.7}{4.0}) = 1.025$$

$$s_\gamma = 1 - 0.4\frac{B'}{L} = 1 - 0.4\,(\frac{1.7}{4.0}) = 0.83$$

$$d_c = 1 + 0.2\,(D_f/B')\tan(45^\circ + \phi/2) = 1 + 0.2\,(\frac{1.5}{1.7})\tan 57.5^\circ = 1.28$$

$$d_q = d_\gamma = 1 + 0.1\,(D_f/B')\tan(45^\circ + \phi/2) = 1 + 0.1\,(\frac{1.5}{1.7})\tan 57.5^\circ = 1.14$$

$$i_c = i_q = (1 - \frac{\alpha}{90})^2 = (1 - \frac{15}{90})^2 = 0.69$$

$$i_\gamma = (1 - \frac{\alpha}{\phi})^2 = (1 - \frac{15}{25})^2 = 0.16$$

Substituting these values,

$$q_{nu} = 15 \times 20.7 \times 1.085 \times 1.28 \times 0.69 + 27 \times 9.7 \times 1.085 \times 1.14 \times 0.69$$
$$+ 0.5 \times 18 \times 1.7 \times 10.9 \times 0.83 \times 1.14 \times 0.16 \times 0.625$$
$$= 297.5 + 223.5 + 15.8 = 536.8 \text{ kN/m}^2$$

Example 15.8 A chimney, with a rigid base 2.5 m square, is placed at a depth of 1 m below the ground surface. The soil is clay with an unconfined compressive strength of 60 kN/m^2 and unit weight of 20 kN/m^3. The weight of the chimney is 60 kN. The chimney has a resultant wind load of 19.5 kN acting parallel to one of the sides of the chimney base at a height of 1.5 m above the ground surface. Determine the factor of safety with respect to bearing capacity. Use Meyerhof's recommendations.

Solution:

The horizontal wind load will have the effect of introducing both inclination and eccentricity of loading. The resultant of the wind load and weight force will be inclined at an angle α to the vertical.

$$\tan\alpha = \frac{\text{Horizontal wind force}}{\text{Vertical weight force}} = \frac{19.5}{60} = 0.325$$

$$\alpha = 18^\circ$$

Height of horizontal load above the base = 1.5 + 1 = 2.5 m. Eccentricity of the resultant load, e can be calculated from,

$$\frac{e}{2.5} = \tan\alpha = 0.325$$

$$e = 2.5 \times 0.325 = 0.81 \text{ m}$$

Reduced dimension B' on account of eccentricity of loading is given by

$$B' = B - 2e_x = 2.5 - 2 \times 0.81 = 0.88 \text{ m}$$

$$A' = B'L = 0.88 \times 2.5 = 2.2 \text{ m}^2$$

For $\phi = 0, N_c = 5.14, N_q = 1, N_\gamma = 0, \tan(45 + \phi/2) = 1$

$$s_c = 1 + 0.2\frac{B'}{L}\tan^2(45 + \phi/2)$$

$$= 1 + 0.2 \times \frac{0.88}{2.5} = 1.07$$

$$s_q = 1$$

$$d_c = 1 + 0.2\frac{D}{B'}\tan(45 + \phi/2)$$

$$= 1 + 0.2 \times \frac{1}{0.88} = 1.23$$

$$d_q = 1$$

$$i_c = i_q = (1 - \frac{\alpha°}{90}) = (1 - \frac{18}{90}) = 0.64$$

From Eq. 15.39,

$$q_u = cN_c s_c d_c i_c + q N_q s_q d_q i_q, \text{ since } N_\gamma = 0$$

$$c_u = \frac{60}{2} = 30 \text{ kN/m}^2$$

Hence $q_u = (30 \times 5.14 \times 1.07 \times 1.23 \times 0.64) + (20 \times 1 \times 1 \times 1 \times 0.64)$

$$= 129.9 + 12.8 = 142.7 \text{ kN/m}^2$$

$$Q_u = q_u A' = 142.7 \times 2.2 = 314 \text{ kN}$$

Factor of safety $= \frac{314}{60} = 5.2$

Example 15.9 A footing 4 m × 2 m in plan, transmits a pressure of 150 kN/m^2 on a cohesive soil having $E = 6 \times 10^4$ kN/m^2 and $\mu = 0.50$. Determine the immediate settlement of the footing at the centre, assuming it to be (a) a flexible footing and (b) a rigid footing.

Solution:

From Eq. 15.64,

$$S_i = qB\left(\frac{1-\mu^2}{E}\right)I_f$$

For $\frac{L}{B} = \frac{4}{2} = 2$, from Table 15.10, $I_f = 1.52$ for a flexible footing and 1.20 for a rigid footing

(a) $S_i = 150 \times 2\left(\frac{1-0.5^2}{6\times10^4}\right)\times 1.52$ m

$= 5.7$ mm

(b) $S_i = 4.5$ mm

If we use the rigidity factor of 0.8 as recommended by IS: 8009, Part I (1976),

$S_{i\,(\text{rigid footing})} = 0.8 \times 5.7 = 4.56$ mm

Example 15.10 Fig. 15.30(a) shows a 2.5 m square footing resting on a sand deposit. The total pressure at foundation level is 200 kN/m^2. The variation of cone penetration resistance with depth is simplified as shown in Fig. 15.30 (b). Determine the settlement of the foundation 6 years after construction. Use the Schmertmann approach. The dash line in Fig. 15.30(b) shows the distribution of the strain-influence factor. The ground water table is deep.

Solution:

From Eq. 15.77,

$$S = C_1 C_2 q \sum_0^{2B} \frac{I_Z}{E}\Delta z.$$

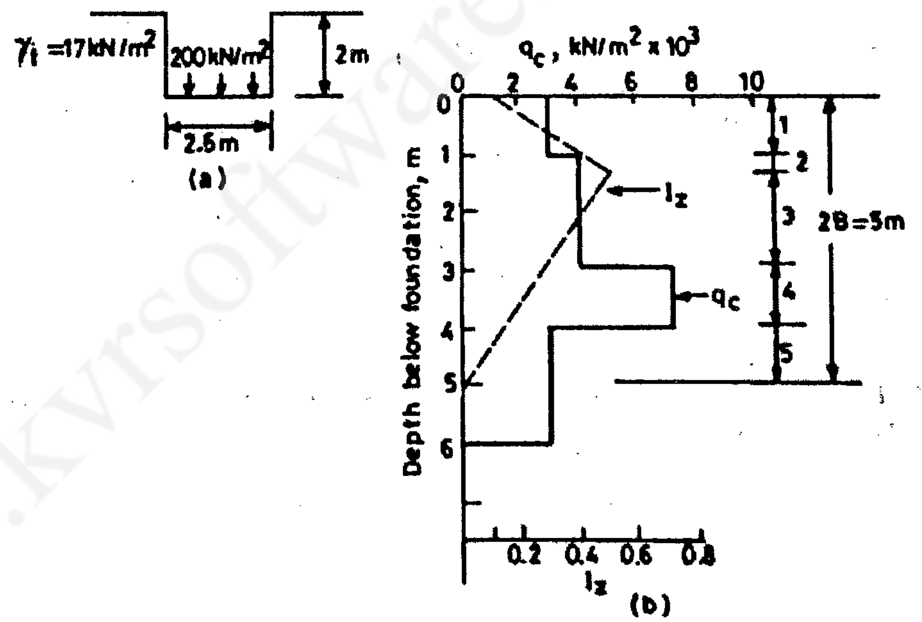


Fig. 15.30 Example 15.10

Here q = net increase in pressure at foundation level

$= 200 - 17 \times 2 = 166$ kN/m^2.

$$C_1 = 1 - 0.5\left(\frac{\bar{\sigma}_0}{q}\right) = 1 - 0.5\left(\frac{17 \times 2}{166}\right) = 0.898$$

$$C_2 = 1 + 0.2 \log_{10} \frac{t}{0.1} = 1 + 0.2 \log \frac{1}{0.1} = 1.356$$

Hence,

$$S = 0.898 \times 1.356 \times 166 \sum_0^{2B} \frac{I_z}{E} \Delta z$$

$$= 202.14 \sum_0^{2B} \frac{I_z}{E} \Delta z$$

The value of $\sum_0^{2B} \frac{I_z}{E} \Delta z$ is determined as shown in the Table below:

Layer	Δz (mm)	q_c ((kN/m^2)	E (2.5 q_c)	I_z	$\frac{I_z}{E} \Delta z$
1	1000	3000	7500	$\frac{0.1 + 0.42}{2} = 0.260$	0.0347
2	250	4000	10000	0.380	0.0095
3	1750	4000	10000	0.385	0.0674
4	1000	7000	17500	0.200	0.0114
5	1000	3000	7500	0.067	0.0089
					Sum = 0.1319

$$S = 202.14 \sum_0^{2b} \frac{I_z}{B} \Delta z$$

$$= 202.14 \times 0.1319$$

$$= 26.7 \text{ mm}$$

Example 15.11 For the conditions shown in Fig. 15.30, determine the settlement of the foundation using the De Beer and Martens approach.

Solution:

From Eq. 15.75,

$$S = 2.3 \frac{H}{C} \log_{10} \frac{\bar{\sigma}_0 + \Delta z}{\bar{\sigma}_0}$$

The seat of the settlement is taken as equal to 2B below the base of the foundation. The calculations are shown in Table below:

Example 15.13 The following data was obtained from a plate load test carried out on a 60 cm square test plate at a depth of 2 m below ground surface on a sandy soil which extends upto a large depth. Determine the settlement of a foundation 3.0 m × 3.0 m carrying a load of 110 t and located at a depth of 3 m below ground surface.

Load test data—

Load intensity, t/m²	5	10	15	20	25	30	35	40
Settlement, mm	2.0	4.0	7.5	11.0	16.3	23.5	34.0	45.0

Water table is located at a large depth from the ground surface.

Solution:

The load-settlement curve is shown in Fig. 15.31

Load intensity on the foundaton $= \frac{110}{3 \times 3} = 12.2$ t/m².

From Fig. 15.31, settlement of the test plate, S_p corresponding to a load intensity of 12.2 t/m² is 5 mm.

From Eq. 15.66,

$$S_f = S_p\left[\frac{B_f(B_p + 30)}{B_p(B_f + 30)}\right]^2$$

$$= 5\left[\frac{300\,(60 + 30)}{60\,(300 + 30)}\right]^2 = 9.3 \text{ mm}$$

The effect of embedment must now be taken into account. Depth of embedment D is equal to the depth of foundation measured from the level at which the test plate is placed.

Thus, $D = 3.0 - 2.0 = 1$ m.

Using Fox's depth factor: For $\frac{D}{\sqrt{(LB)}} = \frac{1}{\sqrt{(3 \times 3)}} = 0.33$ and $\frac{L}{B} = 1$,

Depth correction factor = 0.91 (Fig. 15.18)

Actual settlement of foundation = 0.91 × 9.3 = 8.5 mm

Using Ramasamy, Rao and Prakash's recommendation,

Depth correction factor $= \left[\frac{1}{1 + (D/B)}\right]^{0.5} = \left[\frac{1}{1 + 0.33}\right]^{0.5} = 0.87$

Actual settlement of foundation = 0.87× 9.3 = 8.1 mm.

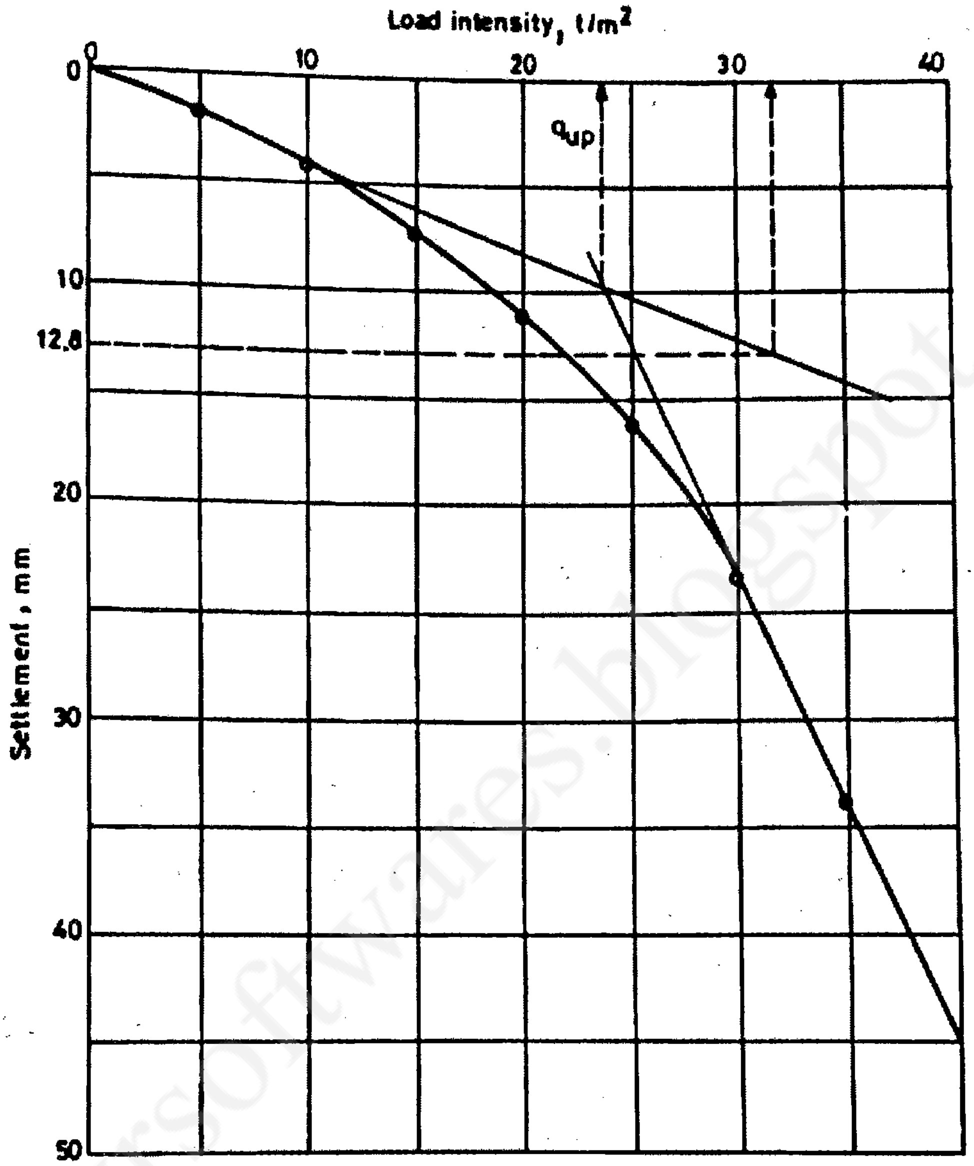


Fig. 15.31 Example 15.13

Example 15.14 Using the load test data in Ex. 15.13, determine the allowable load on a 1.5 m × 1.5 m column footing with its base at a depth of 2 m. The permissible settlement for the foundation is 20 mm and a minimum factor of safety of 3 is required against shear failure. The unit weight of the soil determined at the base of the test pit by the core cutter method was 2.0 t/m³.

Solution:

The angle of shearing resistance of the soil can be worked back with the help of ultimate bearing capacity of the test plate, q_{up}.

$q_{up} = 0.4\ \gamma\ B_p\ N_\gamma$ since there is no surcharge on the test plate.

$$q_{np} = 0.14 \times (23 - 3) \times \left(\frac{3 + 0.3}{6}\right)^2 \times 0.75 \times 1.5 \times 40$$

$$= 38.1 \text{ t/m}^2$$

Hence, the net allowable bearing pressure is

$$q_{a-net} = 38 \text{ t/m}^2, \text{ with settlement criterion governing design.}$$

Example 15.17 Determine the net allowable bearing pressure of the footing in Ex. 15.15, using the IS Code recommendations.

Solution:

$N_{cor} = 18$, from Ex. 15.15

From Fig. 15.12, for $N = 18$, $\phi = 33°$

From Fig. 15.13, for $\phi = 33°$, $N_q = 22$, $N_\gamma = 28$

From Eq. 15.54, for $c' = 0$,

$$q_{nu} = q\,(N_q - 1)\, s_q\, d_q + 0.5\,\gamma B N_\gamma s_\gamma d_\gamma W'$$

$$q = 1.8 \times 1.5 = 2.7 \text{ t/m}^2$$

$$W' = 0.75 \text{ (Fig. 15.10)}$$

From Table 15.7,

$$s_q = 1 + 0.2\frac{B}{L} = 1 + 0.2 \times \frac{3}{5} = 1.12$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4 \times \frac{3}{5} = 0.76$$

$$d_q = d_\gamma = 1 + 0.1\frac{D_f}{B}\tan\left(45° + \frac{\phi}{2}\right)$$

$$= 1 + 0.1 \times \frac{1.5}{3.0}\tan\left(45° + \frac{33}{2}\right)$$

$$= 1.09$$

$$q_{nu} = 2.7 \times 21 \times 1.12 \times 1.09 + 0.5 \times 1.8 \times 3 \times 28 \times 0.76 \times 1.09 \times 0.75$$

$$= 116 \text{ t/m}^2$$

$$q_{ns} = \frac{116}{3} = 38.7 \text{ t/m}^2$$

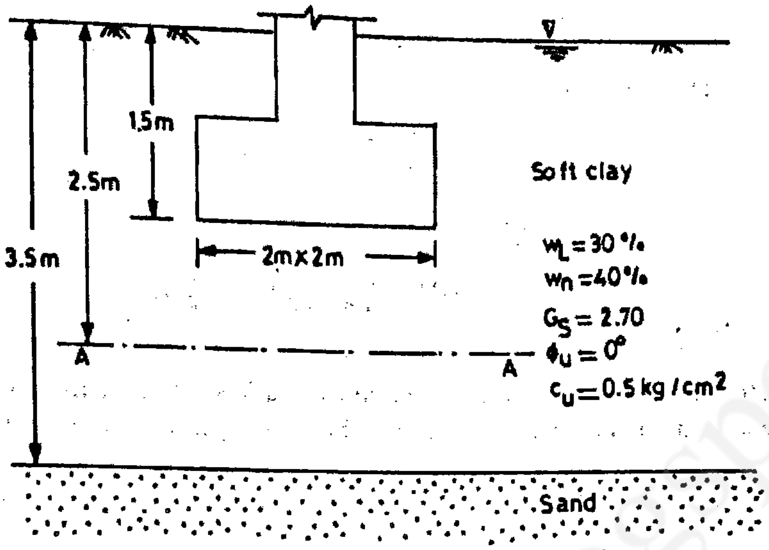


Fig. 15.33 Example 15.19

$$N_c = 5.0\,(1 + 0.2\,D_f/B)\,(1 + 0.2\,B/L)$$
$$= 5.0\,(1 + 0.2 \times 0.75)\,(1 + 0.2 \times 1)$$
$$= 6.9$$
$$q_{nu} = c_u\,N_c = 5 \times 6.9 = 34.5 \text{ t/m}^2$$
$$q_{ns} = \frac{34.5}{3} = 11.5 \text{ t/m}^2$$
$$\gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w \text{ and } e = w_n\,G_s$$
$$\gamma_{sat} = \left(\frac{2.7 + 0.4 \times 2.7}{1 + 0.4 \times 2.7}\right)1 = 1.82 \text{ t/m}^3$$
$$\gamma' = 1.82 - 1.00 = 0.82 \text{ t/m}^3$$

For a normally consolidated clay,

$$S_c = H \times \frac{C_c}{1 + e_0}\log_{10}\frac{\overline{\sigma}_o + \Delta\sigma}{\overline{\sigma}_0}$$
$$C_c = 0.009\,(w_L - 10) \text{ for a normally consolidated clay}$$
$$= 0.009\,(30 - 10)$$
$$= 0.18$$

Effective pressure due to overburden at *AA* (Fig. 15.33),

$$\bar{\sigma}_0 = 0.82 \times 2.5$$
$$= 2.05 \text{ t/m}^2$$

Assuming a load spread of 2 : 1,

$$\Delta\sigma = \frac{11.5 \times 2^2}{3^2} = 5.1 \text{ t/m}^2$$
$$e_0 = w_n G_s = 0.4 \times 2.7 = 1.08$$
$$S_c = 2000 \times \frac{0.18}{1 + 1.08} \log \frac{2.05 + 5.1}{2.05}$$
$$= 94 \text{ mm}$$

Example 15.20. Proportion a square footing to carry a load of 150 t from a column. The depth of foundation is to be kept at 2 m below ground surface. Maximum permissible settlement of the footing is 40 mm and a factor of safety of 3 is required against shear failure. The subsoil is sand with an average corrected N value of 18 as established from borings. Water table is at a large depth. Use Teng's correlations.

Solution:

From Eq. 15.59,

$$q_{nu} = \frac{1}{30}[N^2 B R_w + 3(100 + N^2) D_f R_w]$$
$$= \frac{1}{30}[18^2 \times B \times 1 + 3(100 + 18^2) \times 2 \times 1]$$
$$= 10.8 B + 84.8$$

Total net load, $Q_n = q_{ns} \times B^2 = \frac{q_{nu}}{3} \times B^2$

Hence, $120 = 3.6 B^3 + 28.3 B^3$

$$\Rightarrow B = 1.9 \text{ m.}$$

From Eq. 15.81,

$$q_{np} = 0.14(N - 3)\left(\frac{B + 0.3}{2B}\right)^2 R'_w C_D S_a$$

$$C_D = 1 + \frac{D}{B} = 1 + \frac{2}{1.9} > 2; \text{ hence take } C_D = 2$$

$$q_{np} = 0.14 \times 15 \times \left(\frac{2.2}{3.8}\right)^2 \times 1 \times 2 \times 40$$

$$= 56.13 \text{ t/m}^2$$

$$q_{np} \times B^2 = 56.3 \times 1.9^2 = 203 \text{ t} > 120 \text{ t}$$

Hence, adopt 1.9 m × 1.9 m square footing.

7. The following observations relate to a plate load test conducted on a 30 cm square test plate placed at a depth of 1.5 m in a cohesionless soil deposit:

Intensity of load (kg/cm^2):	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Settlement (mm)	0	2.0	4.0	7.0	11.0	16.0	23.0	32.0	45.0

Plot the load-settlement curve:

(a) Determine the allowable bearing pressure for a 2 m square footing for a minimum factor of safety of 2.5 with respect to shear failure and a maximum permissible settlement 25 mm. Water table is at a depth of 2 m below ground surface. What will be the actual factor of safety with respect to shear failure at the allowable bearing pressure?

(b) If the water table reaches upto the ground surface due to unanticipated reasons, what will be the new value of factor of safety? What settlement can be expected now?

(a) 133 kN/m^2; (settlement governing allowable bearing pressure), F.S = 10.6 (b) 6.8 ; 39 mm.

8. Standard penetration tests were carried out in boreholes at a site and the following *N* values were recorded:

Depth (m)	BH 1	BH 2	BH 3	BH 4
0.75	6	8	5	4
1.50	7	10	9	9
2.25	10	9	8	11
3.00	13	12	10	11
3.75	15	12	12	16
4.50	18	16	12	14

Water table was at a depth of 4 m.
The following loads are to be carried by isolated column footings at a depth of 1 m in a sand soil with $\gamma_t = 17\ kN/m^3$—500 kN, 700 kN and 1200 kN.
Proportion the footings so that the settlement does not exceed 40 mm. Use the Peck, Hanson, Thornburn method. Check for safety against shear failure in each case.

(For $P = 700$ kN, a footing 2 m square will be O.K.)

9. A footing 2.0 m × 2.0 m is located at a depth of 1.5 m in a sand deposit. Borings have indicated that the average corrected *N* value at the site is 25. Water table is at a depth of 2 m below the ground surface. Determine the net allowable bearing pressure for a factor of safety of 3 against shear failure and a permissible settlement of 25 mm. Use Teng's equations. ($q_{a-net} = q_{np} \cong 278\ kN/m^2$)

10. A footing 3 m × 3 m is placed at a depth of 1.5 m in a stratum of saturated clay extending upto a depth of 10 m. Water table is quite close to the ground surface. The following parameters have been determined for the clay: $c_u = 100\ kN/m^2$, $\phi_u = 0$, $c' = 1.5\ t/m^2$, $\phi' = 25°$,

$$m_v = 6 \times 10^{-4}\ m^2/t,\ \gamma_{sat} = 20\ kN/m^3.$$

15.1 Introduction

Foundations are the sub-structural elements which are in direct contact with the ground and transmits the load of the superstructure to the earth in such a way that neither supporting soil failed in shear nor in excessive settlement.

Broadly, foundations may be grouped as shallow or deep foundation depending on the depth of installation of foundation.

15.2 Modes of Failure of a Structure

A structure when loaded may fail in following two ways:

(i) **Failure due to shear :** When the load supporting power of soil is lesser than structural load at the foundation level.

(ii) **Failure due to excessive settlement :** When the settlement in the soil exceed the tolerance limit. In particular, differential settlements should not cause any unacceptable damage which may interfere with the function of the structure.

15.3 Bearing Capacity

The load carrying capacity of foundation soil or rock which enables it to bear and transmit loads from a structure is know as bearing capacity.

15.4 Factors Affecting Bearing Capacity

1. Nature of soil and its physical and engineering properties.
2. Nature of foundation and other details such as the size, shape, depth and rigidity of the structure.
3. Location of the ground water table relative to the foundation level.
4. The total and differential settlements that the structure can with stand without functional failure.
5. Initial stresses condition of the soil due to prehistory or due to the existing structure near proposed foundation.
6. Nature and type of loading i.e. centric or eccentric.
7. Mode of shear failure i.e. General, Local or Punching shear failure.

15.5 Important Definitions

1. **Gross pressure intensity (q_g):** It is the total pressure at the base of foundation due to the weight of super structure, self weight of footing and weight of the earth fill.

Note: The gross pressure intensity at the time of failure is nothing but the ultimate bearing capacity of the soil. In general, gross pressure intensity may be greater, equal or smaller than bearing capacity.

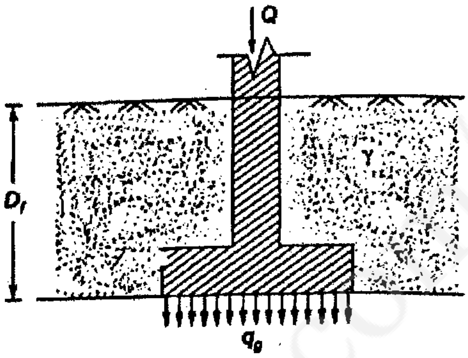


Fig. 15.1

2. **Net pressure intensity (q_n):** It is that part of gross pressure at the base of footing which is in excess to initial effective overburden pressure.

$$q_n = q_g - \gamma D_f$$

3. **Ultimate bearing capacity (q_u):** Ultimate bearing capacity is defined as the minimum gross pressure at the base of the foundation at which the soil just fails in shear.

OR

The maximum gross pressure, which a foundation can with stand without the occurrence of shear failure.

4. **Net ultimate bearing capacity (q_{nu}):** It is that maximum net pressure which can be applied at the base of foundation without shear failure.

OR

It is the minimum net pressure at which the soil just fails in shear.

$$q_{nu} = q_u - \bar{\sigma} = q_u - \gamma D_f$$

5. **Net safe bearing capacity (q_{ns}):** It is that net pressure which can be applied safely at the base of footing without the risk of shear failure.

$$q_{ns} = \frac{q_{nu}}{F} = \frac{q_u - \gamma D_f}{F}$$

Generally F is taken between 2 to 3.

6. **Safe bearing capacity (q_s):** It is that gross pressure at the base of footing which can be applied safely without risk of shear failure.

$$q_{safe} \text{ or } q_s = q_{ns} + \bar{\sigma}$$

$$= \frac{q_u - \bar{\sigma}}{F} + \bar{\sigma}$$

15.6 Mode of Shear Failure

Three principal modes of shear failure have been identified based on the model test of strip footings on sand.

1. **General shear failure**
   - General shear failure, usually associated with medium to dense or stiff soils of relatively low compressibility.

- At the time of failure, most of the soil within stress zone reaches in plastic state except central portion.
- Main characteristics of shear failure are :
  (a) A well defined slip surface developed on both or one side of the footing.
  (b) A sudden, catastrophic failure accompanied by tilting of foundation.
  (c) Bulging of ground surface adjacent to the foundation before failure.

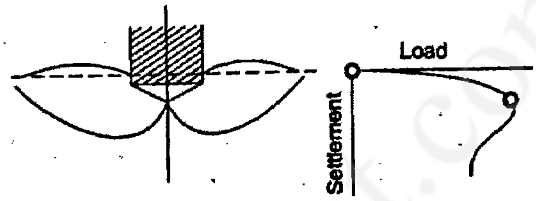


Fig. 15.2 General shear failure

2. **Local shear failure**
   - Local shear failure is an intermediate failure mode characterized by well defined slip surface immediately below the footing but extending only a short distance into the soil mass, as shown in figure.
   - Since, the stress zone does not extend upto ground level. Therefore, only little bulging of soil around the footing.

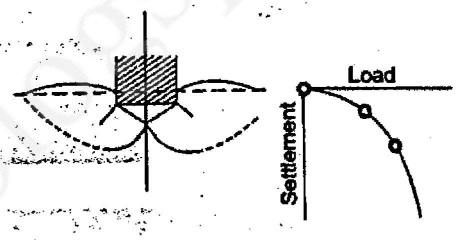


Fig. 15.3 Local shear failure

   - Main characteristics of local shear failure are :
     (a) Occurs in loose sands and soft clays in shallow foundations.
     (b) Well defined wedge and slip surface only below the footing.
     (c) Slip surface not visible beyond the edges of the foundation.
     (d) Soil below footing is more stressed as compared to adjacent soil.

3. **Punching shear failure**
   - Punching shear failure, usually associated with pile footings in loose sand or soft clays.
   - The soil below the foundation is highly stressed which gets separated by vertical shear from adjacent soil. Hence, adjacent soil will remain unstressed.
   - Main characteristics of punching shear failure are :
     (a) Poorly defined failure planes
     (b) Soil zone beyond the footing is not affected or little affected
     (c) There are no tilting and heaving of adjacent ground but large settlement are recorded.

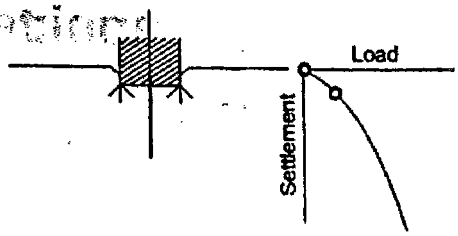


Fig. 15.4 Punching shear failure

Comparison between General and Local Shear Failure

1. For sands

Parameter	General Shear Failure	Local Shear Failure
(i) Angle of shear friction (ϕ)	> 36°	< 28°
(ii) Void ratio (e)	< 0.55	> 0.75
(iii) Relative Density (I_D)	> 70%	< 30%
(iv) Standard Penetration value (N)	> 30	< 5

2. For c-ϕ soils
   (i) General shear failure occurs at a low strain i.e. <5%.
   (ii) Local shear failure occurs at a large strain i.e. 10 to 20%.
3. For Pure clays
   (i) For purely cohesive clay, general shear failure may be assumed to occur with unconfined compressive strength $q_u \geq 100$ kN/m².
   (ii) Local shear failure occurs in soft to medium clays with unconfined compressive strength $q_u < 80$ kN/m²

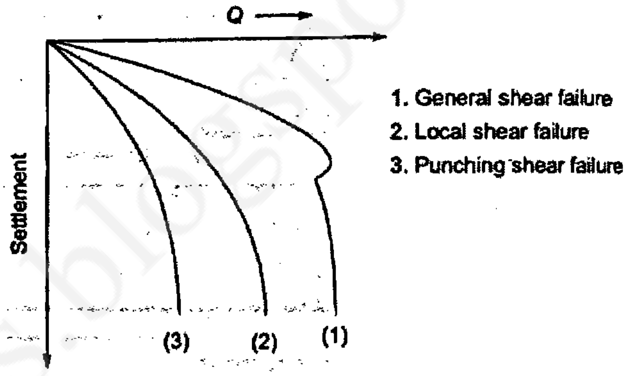


Fig. 15.5 Load-settlement curve for soils

15.7 Methods to Determine Bearing Capacity

1. Analytical Methods
   These methods are based on soil and foundation properties.
   These are divided into three groups:
   (a) Based on theory of elasticity
   (i) Schleicher's method
   (b) Based on classical earth pressure theory
   (i) Rankine's theory (only for ϕ-soils)
   (ii) Pauker's theory (only for ϕ-soils)
   (iii) Bell's theory (for c-ϕ soils)
   (c) Based on plastic theory
   (i) Fellenius method (only for c-soils)
   (ii) Prandtl's method (for c-ϕ soils)
   (iii) Terzaghi's method (for c-ϕ soils)
   (iv) Meyerhoff's method (for c-ϕ soils)
   (v) Skempton's method (only for c-soils)
   (vi) Brink-hansen's method (for c-ϕ soils)
   (vii) Balla's method (for c-ϕ soils)
   (viii) Vesic's method (for c-ϕ soils)
   (ix) I.S. code method

2. **Codal Provisions**
   - These methods are based on various building codes like BIS, IRC, CPWD, etc. on the basic of soil testing results, they have published bearing capacity of zonal soils.
   - These bearing capacity values can be used for small scale works.
3. **Field methods**
   (i) Standard penetration test (SPT)
   (ii) Cone penetration test (CPT)
   (iii) Plate load test (PLT)

15.8 Analytical Methods

15.8.1 Schleicher's Method

- This is based on theory of elasticity and Boussinesq's stress distribution.
- Elastic settlement 'S' of the soil directly underneath a perfectly elastic bearing slab is given by

$$S = K \cdot q\sqrt{A}\frac{(1-\mu^2)}{E}$$

Where K = Shape coefficient or influence value, depending on size, shape and rigidity of the slab.
q = Net pressure applied from the slab on to the soil
A = Area of bearing slab
E = Modulus of elasticity of soil
μ = Poisson's ratio of the soil

- If 'S_1' and 'S_2' are settlements brought about by two bearing areas of similar shape of different sizes, 'A_1' and 'A_2' respectively with equal contact pressure.

then, $$\frac{S_1}{S_2} = \sqrt{\frac{A_1}{A_2}}$$

Note: It should be noted that maximum settlement occurs at the centre of circular and rectangular bearing areas and a minimum value occurs at the periphery in clays.

15.8.2 Rankine's Theory

- Rankine's considered the plastic equilibrium of two adjacent soil element, one immediately beneath the footing say element '1' and other just beyond the edge of footing say element '2' as show in figure below.

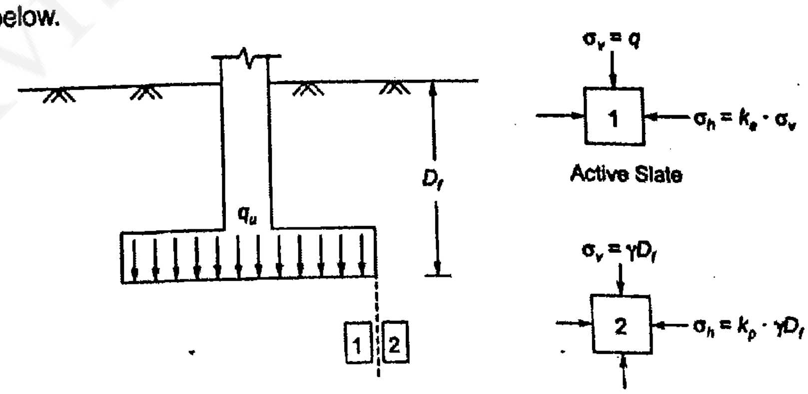


Fig. 15.6

- The vertical pressure on element 1 is equal to 'q_u' whereas on element '2' it is γD_f.
- Element '1' is in active state. Hence, the vertical stress is the major principal stress and the lateral stress is the minor principal stress. Whereas, element '2' is in passive state. Hence lateral stress becomes the major stress, and the vertical stress becomes the minor principal stress.
- For element '2',

$$\sigma_h = k_p \sigma_v = \frac{1+\sin\phi}{1-\sin\phi} \cdot \gamma D_f \quad \text{... (i)}$$

For element 1,

$$\sigma_h = k_a \sigma_v = k_a q_u$$

or

$$q_u = \frac{\sigma_h}{k_a} = k_p \sigma_h = \frac{1+\sin\phi}{1-\sin\phi} \cdot \sigma_h \quad \text{... (ii)}$$

From equation (i) and (ii),

$$q_u = \left[\frac{1+\sin\phi}{1-\sin\phi}\right]^2 \cdot \gamma D_f$$

$$q_u = k_p^2 \gamma D_f \quad \text{or} \quad \tan^4\left(45+\frac{\phi}{2}\right)\gamma D_f$$

Limitations:

1. Above formula gives the bearing capacity of cohesionless soil as zero at the ground surface ($D_f = 0$) but practically it is not zero.
2. According to this formula, there is no effect of size and shape of footing on ultimate bearing capacity which is incorrect.
3. This theory is not applicable for c-soils and c-ϕ soils.

From Fig. 15.31, $q_{up} = 24$ t/m^2, from the double tangent method.

$$N_\gamma = \frac{24}{0.4 \times 2 \times 0.6} = 50$$

From Fig. 15.12, for $N_\gamma = 50$, $\phi = 36.5°$ and $N_q = 40$.

For the foundation,

$$q_{nu} = \gamma D_f (N_q - 1) + 0.4\, \gamma B_f N_\gamma$$

$$= 2 \times 2 \times 39 + 0.4 \times 2 \times 1.5 \times 50$$

$$= 216 \text{ t/m}^2$$

$$q_{ns} = \frac{216}{3} = 72 \text{ t/m}^2$$

$$S_p = S_f \left[\frac{B_p (B_f + 30)}{B_f (B_p + 30)}\right]^2$$

$$= 20 \left[\frac{60\,(150 + 30)}{150\,(60 + 30)}\right]^2 = 12.8 \text{ mm}$$

From Fig. 15.31, the load intensity corresponding to a settlement of 12.8 mm may be determined from the method suggested by Rao and Ramasamy.

Net safe bearing pressure = 32 t/m^2.

Hence, settlement criterion governs the design and the net allowable bearing pressure = 32 t/m^2.

Allowable load on the footing = $32 \times 1.5^2 = 72$ t.

Example 15.15. Determine the allowable bearing pressure for the rectangular footing shown in Fig. 15.32 using Peck, Hanson and Thornburn procedure. Allowable settlement = 40 mm.

Solution:

The average value of N_{cor} between El − 1.50 m and El − 6.00 m will be considered for calculations. This would mean a depth equal to 1.5 *B* or 4.5 m below the base. If one had the choice, one would include the values upto a depth equal to 2 *B* below the base.

The field values of *N*, that is N_f, are corrected first for overburden pressure and then for dilatancy (where appropriate) using the Peck, Hanson and Thornburn procedure using Eq. 19.6 or Fig. 19.6 and Eq. 19.9.

It may be noted that dilatancy correction is not required in the coarse sand layer.

$$\text{Average } N_{cor} = \frac{14.4 + 19.5 + 18.1 + 20.9 + 18.8 + 16.3 + 20.9}{7} = 18$$

From Fig. 15.23, for $N = 18$, $B = 3$ m,

Settlement for a pressure of 1 kg/cm^2 or 10 t/m^2 is 0.015 m or 15 mm.

Taking into account the position of water table,

$$\text{Actual settlement} = \frac{15}{\text{correction factor}} = \frac{15}{0.75} = 20 \text{ mm}$$

For $$S_a = 40 \text{ mm}, q_{np} = \frac{10}{20} \times 40 = 20 \text{ t/m}^2$$

Hence, $$q_{a-net} = 20 \text{ t/m}^2.$$

Example 15.18 Determine the net safe bearing pressure for the footing in Ex. 15.15 using Meyerhof's correlation.

Solution:

From Eq. 15.86, for $B > 1.2$ m,

$$q_{np} = 0.032\, N\, R_{D_2} \left(\frac{B+0.3}{B}\right)^2 S_a$$

$$R_{D_2} = 1 + 0.33\frac{D}{B} = 1 + 0.33 \times \frac{1.5}{3} = 1.16$$

N_{cor} is taken as in Teng's recommendation, that is, 23.

$$q_{np} = 0.032 \times 23 \times 1.16 \left(\frac{3+0.3}{3}\right)^2 \times 40$$

$$= 41.3 \text{ t/m}^2$$

Effect of water table is not included in Meyerhof's correlation.

Example 15.19 A footing, 2 m square, rests on a soft clay soil with its base at a depth of 1.5 m from ground surface. The clay stratum is 3.5 m thick and is underlain by a firm sand stratum. The clay soil has the following properties:

$$w_L = 30\%;\ w_n = 40\%;\ G_s = 2.70;\ \phi_u = 0;\ c_u = 0.5 \text{ kg/cm}^2$$

It is known that the clay stratum is normally consolidated. Using Skempton's equation, determine the net safe bearing capacity of the footing. Compute the settlement that would result if this load intensity were allowed to act on the footing. Natural water table is quite close to the ground surface.

Solution:

$$D_f/B = \frac{1.5}{2} = 0.75 < 2.5$$

Hence from Eq. 15.36,

or $$1.41 + 0.02\,D_f + 1.9\,D_f = 14$$

giving $$D_f = 6.56 \text{ m}$$

Hence, the base of the raft must be placed at a depth of 6.56 m.

In an actual design, the settlement resulting from the net loading intensity of 14 - 1.9 × 6.56, *i.e.*, 2.5 t/m^2 will have to be computed and checked against the permissible settlement to ensure that the design is safe from the settlement criterion too.

PROBLEMS

1. A strip footing 1.2 m wide, is supported on a soil with its base at a depth of 1 m below ground surface. The soil properties are as below:

$$c' = 15 \text{ kN/m}^2, \phi' = 28°, \gamma_t = 18 \text{ kN/m}^3, \gamma' = 10 \text{ kN/m}^3.$$

Determine the ultimate bearing capacity if
(a) water table is at a great depth
(b) water table is at the level of the base of the footing
(c) water table is at the ground surface.
Use the Terzaghi equation. (a) 990 kN/m^2, (b) 914 kN/m^2 (c) 765 kN/m^2

2. A column, carrying a load of 750 kN, has to be supported by a square footing with its base at 1.5 m depth. What is the required size of the foundation which will provide a factor of safety of 3 against shear failure? Assume $c' = 10$ kN/m^2 $\phi = 30°$.$\gamma_t = 18$ kN/m^3, $\gamma' = 10$ t/m^3 Water table is at 1.5 m depth. Use Terzaghi's equation. ($B = 1.5$ m)

3. A rectangular footing, 2 m × 3.5 m, is placed at a depth of 1.5 m below ground surface. Determine both by Meyerhof's recommendations as well as IS: 6403 (1981) recommendations, the net safe load that can be supported by the footing with a factor of safety of 2.5 with respect to shear failure. The soil properties are: $c' = 20$ kN/m^2, $\phi' = 22°$. (a) 2424 kN, (b) 2251 kN.

4. A retaining wall, 6 m high, has a vertical back and supports a horizontal sand backfill with $\phi' = 30°$ and $\gamma = 17$ kN/m^3. It is resting on a soil which has $\gamma = 17.5$ kN/m^3, $c' = 0$ and $\phi' = 32°$. The natural ground level is located 1.5 m above the toe level. The weight of the wall is 250 kN/m and its line of action passes at a distance of 0.75 m from the heel. The base width of the retaining wall is 2.5 m.
Determine the ultimate bearing capacity per metre length of the wall, using IS: 6403 (1981) recommendations.

5. A load of 400 kN/m is carried by a strip footing, 2 m wide, located at a depth of 1.5 m in a clay soil with $\gamma_{sat} = 20$ kN/m^3. Water table is quite close to the ground surface. Determine the factor of safety with respect to shear failure (a) when $c_u = 10$ kN/m^2 and $\phi_u = 0$ and (b) when $c' = 15$ kN/m^2 and $\phi' = 28°$ (a) 2.85 (b) 4.5

6. A 30 cm square test plate settles by 18 mm in a plate load test conducted on a granular soil when the loading intensity was 200 kN/m^2. Estimate the likely settlement in a footing 1.5 m square, resting on the same soil, at the same intensity of loading. (50 mm)